Chapter: 3

Pair Of Linear Equations In Two Variables

Exercise : 3.1

1. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting ?) Represent this situation algebraically and graphically.

Sol:- Let Aftab's ~~real~~ present age be 'x'
& Aftab's daughter's present age be 'y'
Then,
7 years ago,
Aftab's age = $(x-7)$
Aftab's daughter's age = $(y-7)$
Now, According to Problem,

$$x-7 = 7(y-7)$$
$$x-7 = 7y-49$$
$$x = 7y-49+7$$
$$x = 7y-42$$
$$x-7y = -42 \quad \text{———} \ (i)$$

Also,
3 years later,
Aftab's age = $x+3$
Aftab's daughter's age = $y+3$
Now,
According to Problem,

$$x+3 = 3(y+3)$$

$$x + 3 = 3y + 9$$

$$x = 3y + 9 - 3$$

$$x = 3y + 6$$

$$x - 3y = 6 \quad \text{——— (ii)}$$

Now,

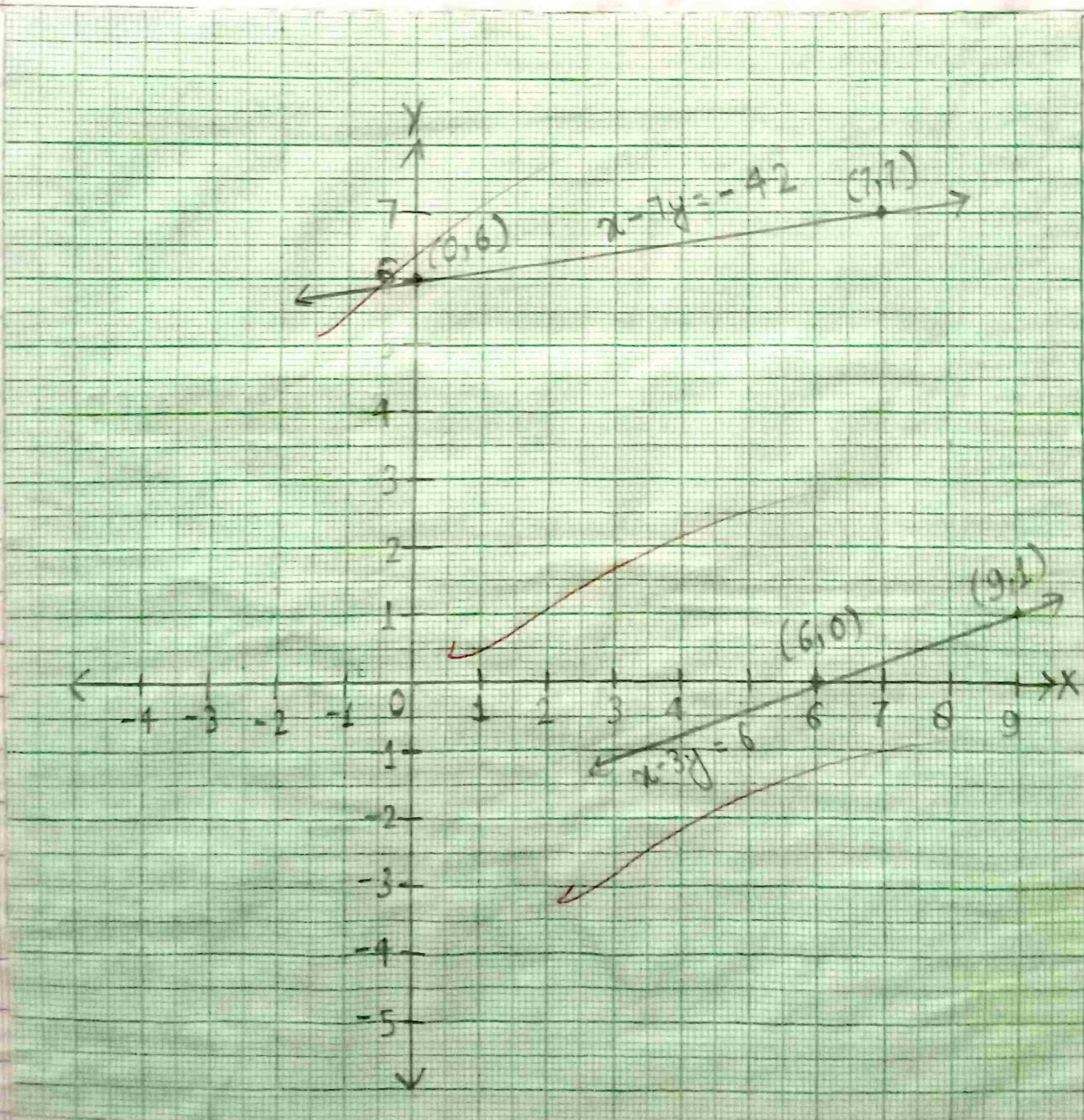
In tabular representation

x	0	7
y	6	7

eqⁿ (i)

x	6	9
y	0	1

eqⁿ (ii)



2. The coach of a cricket team buys 3 bats and 6 balls for ₹3900. Later, she buys another bat and 3 more balls of the same kind for ₹1300. Represent this situation algebraically and graphically.

Sol:- Let the cost of bat be 'x'
& the cost of ball be 'y'

In Ist case: $3x + 6y = 3900$

$$3(x + 2y) = 3900$$

$$x + 2y = \frac{3900}{3}$$

$$x + 2y = 1300 \quad \text{—— (i)}$$

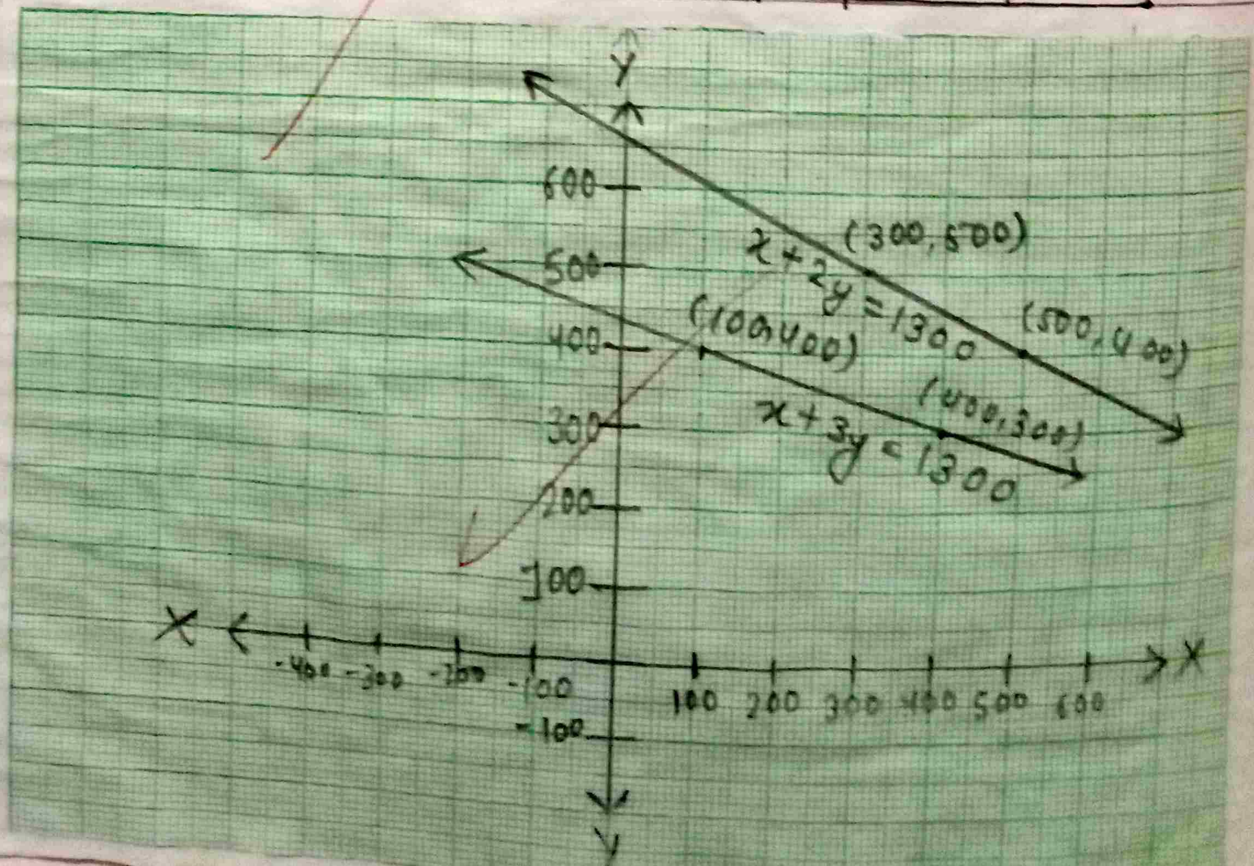
In IInd case: $x + 3y = 1300$ —— (ii)

Now,

In tabular representation

x	300	500
y	500	400

x	100	400
y	400	300



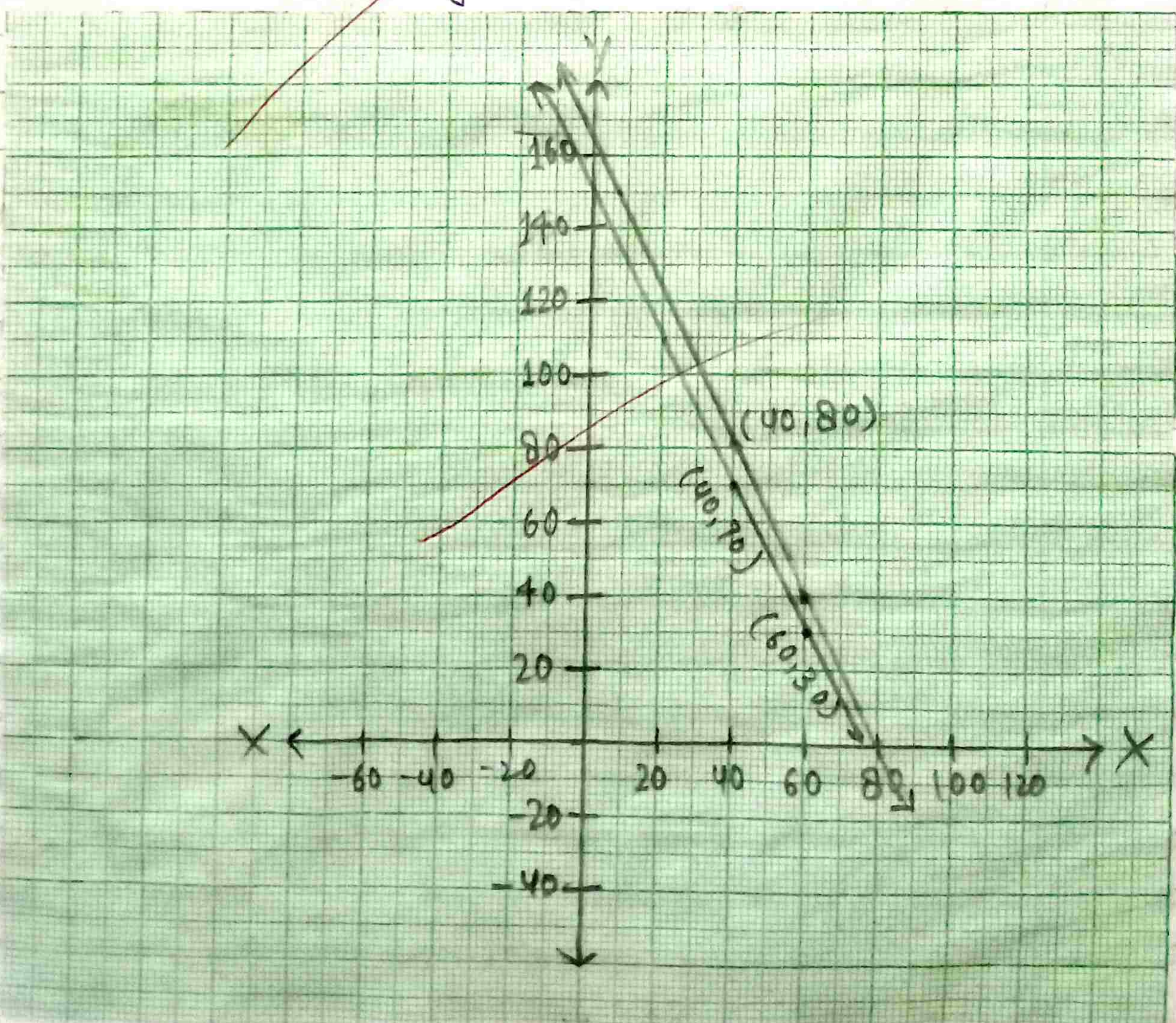
3. The cost of 2 kg of apples and 1 kg of grapes on a day was found to be ₹160. After a month, the cost of 4 kg of apples and 2 kg of grapes is ₹300. Represent the situation algebraically and geometrically.

Sol:- Let the cost of apples be 'x'
& the cost of grapes be 'y'

In Ist Case: $2x + y = 160$ ——— (i)

In IInd Case: $4x + 2y = 300$
$2(2x + y) = 300$
$2x + y = \frac{300}{2}$

$2x + y = 150$ ——— (ii)



Exercise : 3.2

1. From the pair of linear equations in the following problems, and find their solutions graphically

(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than boys, find the ~~solution~~ number of boys and girls who took part in that quiz.

Sol:- Let the number of boys be 'x'
& the number of girls be 'y'

In Ist Case: $x + y = 10$ —— (i)

In IInd Case: $x + 4 = y$
$x - y = -4$ —— (ii)

Solution of eqn (i) and (ii) are:

x	6	4
y	4	6

x	0	-4
y	4	0

(iii) 5 pencils and 7 pens together cost ₹50, whereas 7 pencils and 5 pens together cost ₹46. Find the cost of one pencil and that of one pen.

Sol:- Let the number of pencils be 'x'
& the number of pens be 'y'

In Ist Case : $5x + 7y = 50$

In IInd Case : $7x + 5y = 46$

Solution of eqn (i) and (ii) are :

x	3	10
y	5	0

x	3	[illegible]
y	5	-2

2. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident.

(i) $5x - 4y + 8 = 0$; $7x + 6y - 9 = 0$

Sol:- $\frac{a_1}{a_2} = \frac{5}{7}$; $\frac{b_1}{b_2} = \frac{-4}{6}$

Here,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Hence,

~~It~~ Linear equations will intersect at a point

(ii) $9x + 3y + 12 = 0$; $18x + 6y + 24 = 0$

Sol:- $\frac{a_1}{a_2} = \frac{\cancel{9}^1}{\cancel{18}_2} = \frac{1}{2}$; $\frac{b_1}{b_2} = \frac{\cancel{3}^1}{\cancel{6}_2} = \frac{1}{2}$; $\frac{c_1}{c_2} = \frac{\cancel{12}^1}{\cancel{24}_2} = \frac{1}{2}$

Here,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence,

Linear equation will be ~~coindic~~ coincident

(iii) $6x - 3y + 10 = 0$; $2x - y + 9 = 0$

Sol:- $\frac{a_1}{a_2} = \frac{\cancel{6}^3}{\cancel{2}_1} = \frac{3}{1}$; $\frac{b_1}{b_2} = \frac{+3}{+1}$; $\frac{c_1}{c_2} = \frac{10}{9}$

Here,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence,

Linear Equation will be parallel.

3. On ~~co~~ comparing the ratios $\frac{a_1}{b_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

(i) $3x + 2y = 5$; $2x - 3y = 7$

And: $3x + 2y - 5 = 0$; $2x - 3y - 7 = 0$

$$\frac{a_1}{a_2} = \frac{3}{2} , \frac{b_1}{b_2} = \frac{2}{-3}$$

Here,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Hence,

Linear equation is consistent

(ii) $2x - 3y = 8$; $4x - 6y = 9$

Sol: $2x - 3y - 8 = 0$; $4x - 6y - 9 = 0$

$$\frac{a_1}{a_2} = \frac{\cancel{2}^1}{\cancel{4}_2} = \frac{1}{2} , \frac{b_1}{b_2} = \frac{\cancel{+3}^1}{\cancel{+6}_2} = \frac{1}{2} , \frac{c_1}{c_2} = \frac{+8}{+9}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence,

Linear Equation is inconsistent

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$; $9x - 10y = 14$

Sol: $\frac{3}{2}x + \frac{5}{3}y - 7 = 0$; $9x - 10y - 14 = 0$

$\frac{a_1}{a_2} = \frac{1}{6}$; $\frac{b_1}{b_2} = \frac{-\cancel{5}1}{6}$, $\frac{c_1}{c_2} = \cancel{\frac{+7\,1}{+14\,2}}$

Here,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Hence,

Linear Equation is consistent

(iv) $5x - 3y = 11$; $-10x + 6y = -22$

Sol: $5x - 3y - 11 = 0$; $-10x + 6y + 22 = 0$

$\frac{a_1}{a_2} = \frac{\cancel{5}1}{-\cancel{10}2} = \frac{-1}{2}$, $\frac{b_1}{b_2} = \frac{-\cancel{3}1}{\cancel{6}2} = \frac{-1}{2}$, $\frac{c_1}{c_2} = \frac{-\cancel{11}1}{\cancel{22}2} = \frac{-1}{2}$

Here,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence,

Linear Equation is consistent.

(v) $\frac{4}{3}x + 2y = 8$; $2x + 3y = 12$

Sol:- $\frac{4}{3}x + 2y - 8 = 0$; $2x + 3y - 12 = 0$

$\frac{a_1}{a_2} = \frac{2}{3}$, $\frac{b_1}{b_2} = \frac{2}{3}$, $\frac{c_1}{c_2} = \frac{+\cancel{8}^{2}}{+\cancel{12}_{3}} = \frac{2}{3}$

Here,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence,

Linear Equation is consistent.

4. Which of the following pairs of linear equation are consistent / inconsistent ? If consistent, obtain the solution graphically?

(i) $x + y = 5$; $2x + 2y = 10$

Sol:- $x + y = 5$; $2(x+y) = 10$
$\Rightarrow x + y = 5$

$\frac{a_1}{a_2} = \frac{1}{1}$, $\frac{b_1}{b_2} = \frac{1}{1}$, $\frac{c_1}{c_2} = \frac{\cancel{5}^{1}}{\cancel{5}_{1}} = \frac{1}{1}$

Here,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence,

Linear Equation is Consistent

The tabular representation of eqⁿ (i) and (ii) are:

x	2	3
y	3	2

eqⁿ (i)

x	2	3
y	3	2

eqⁿ (ii)

(ii) $x - y = 8$; $3x - 3y = 16$

Sol: $x - y - 8 = 0$; $3x - 3y - 16 = 0$

$\frac{a_1}{a_2} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{+1}{+3}$, $\frac{c_1}{c_2} = \frac{+\cancel{8}\,1}{+\cancel{16}\,2} = \frac{1}{2}$

Hence,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence,
Linear Equation is Inconsistent

(iii) $2x + y - 6 = 0$; $4x - 2y - 4 = 0$

Sol :- $\frac{a_1}{a_2} = \frac{\cancel{2}\,1}{\cancel{4}\,2} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{1}{-2}$

Here,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Hence,
Linear Equation is Consistent

(iv) $2x - 2y - 2 = 0$; $4x - 4y - 5 = 0$

Sol: $\frac{a_1}{a_2} = \frac{\cancel{2}^1}{\cancel{4}_2} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{-\cancel{2}1}{-\cancel{4}_2} = \frac{1}{2}$, $\frac{c_1}{c_2} = \frac{-2}{-5}$

Here,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence,

Linear Equation is Inconsistent.

5. Half the perimeter of a rectangular garden, whose length is 4m more than its width, is 36m. Find the dimension of the garden.

Sol: Given, Half perimeter of rectangular garden = 36m
Length = ?
Breadth = ?

Let the length of the garden be 'x' and breadth of garden be 'y'

$x = y + 4$
$x - y = 4$ —— (i)

$x + y = 36$ —— (ii)

Adding eq^n (i) & (ii)

$$x - y = 4$$
$$x + y = 36$$
$$2x = 40$$
$$x = \frac{40}{2}$$
$$\boxed{x = 20}$$

Putting value of x in eqⁿ (i)

$$20 - y = 4$$
$$20 - 4 = y$$
$$\boxed{y = 16}$$

Hence,
length of garden = 20m
breadth of garden = 16 m

6. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of pair so formed is :

(i) intersecting lines

Ans: $x - 2y = 0$; $3x + 4y - 20 = 0$

(ii) parallel lines

Ans: $x + 2y - 4 = 0$; $2x + 4y - 12 = 0$

(iii) coincident lines

Ans = $2x + 3y - 9 = 0$; $4x + 6y - 18 = 0$

Methods for solving linear equations

1. Graphical Method :

(i) If the lines intersect at a point, then that point gives the unique solution of the two equations. In this case, the pair of equation is consistent.

(ii) If the lines coincide, then there are infinitely many solutions — each point on the line being a solution. In this case, the pair of equation is dependent (consistent).

(iii) If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is inconsistent.

2. Algebraic Methods :

(i) Substitution Method
(ii) Elimination Method
(iii) Cross-multiplication Method

Exercise : 3.3

1. Solve the following pair of linear equations by the substitution method.

(i) $x + y = 14$; $x - y = 4$

Sol:- In equation (i)

$$x + y = 14$$
$$x = 14 - y$$

Putting value of x in eq^n (ii)

$$14 - y - y = 4$$
$$-2y = 4 - 14$$
$$+2y = +10$$
$$y = \frac{10}{2}$$

$$\boxed{y = 5}$$

Putting value of y in eq^n (i)

$$x + 5 = 14$$
$$x = 14 - 5$$
$$\boxed{x = 9}$$

(ii) $s - t = 3$; $\frac{s}{3} + \frac{t}{2} = 6$

Sol:- In equation (i)

$$s - t = 3$$

$s = 3 + t$

Putting value of s in eqⁿ (ii)

$$\frac{3+t}{3} + \frac{t}{2} = 6$$

$$\frac{6+2t+3t}{6} = 6$$

$$6 + 5t = 6 \times 6$$
$$6+5t = 36$$
$$5t = 36 - 6$$
$$5t = 30$$
$$t = \frac{30}{5}$$

$$\boxed{t = 6}$$

Putting value of t in eqⁿ (i)

$$s - 6 = 3$$
$$s = 3 + 6$$

$$\boxed{s = 9}$$

(iii) $3x - y = 3$; $9x - 3y = 9$

Sol:- In equation (i)

$$3x - y = 3$$
$$3x = 3 + y$$

$$x = \frac{y+3}{3}$$

Putting value of x in eq^n (ii)

$$9\left(\frac{y+3}{3}\right) - 3y = 9$$

$$\frac{9y+27}{3} - 3y = 9$$

$$\frac{9y+27-9y}{3} = 3$$

$$\frac{27}{3} = 3$$

Infinitely many solutions.

(iv) $0.2x + 0.3y = 1.3$; $0.4x + 0.5y = 2.3$

Sol:- In equation (i)

$$0.2x + 0.3y = 1.3$$
$$0.2x = 1.3 - 0.3y$$
$$x = \frac{1.3 - 0.3y}{0.2}$$

Putting value of x in eq^n (ii)

$$0.4\left(\frac{1.3-0.3y}{0.2}\right) + 0.5y = 2.3$$

$$\frac{0.52 - 0.12y}{0.2} + 0.5y = 2.3$$

$$\frac{0.52 - 0.12y + 0.10y}{0.2} = 2.3$$

$$0.52 - 0.2y = 2.3 \times 0.2$$
$$0.52 - 0.2y = 0.46$$
$$-0.2y = 0.46 - 0.52$$
$$+0.2y = +0.6$$
$$y = \frac{\cancel{0.6}\,3}{\cancel{0.2}\,1}$$

$$\boxed{y = 3}$$

Putting value of y in eq^n (i)

$$0.2x + \cancel{0.6x}\ 0.3 \times 3 = 1.3$$
$$0.2x + 0.9 = 1.3$$
$$0.2x = 1.3 - 0.9$$
$$0.2x = 0.4$$
$$x = \frac{\cancel{0.4}\,2}{\cancel{0.2}\,1}$$

$$\boxed{x = 2}$$

(v) $\sqrt{2}x + \sqrt{3}y = 0$; $\sqrt{3}x - \sqrt{8}y = 0$

Sol:- In equation (i)

$$\sqrt{2}x + \sqrt{3}y = 0$$
$$\sqrt{2}x = -\sqrt{3}y$$

$$x = -\frac{\sqrt{3}y}{\sqrt{2}}$$

Putting value of x in eqⁿ (ii)

$$\sqrt{3} \times -\frac{\sqrt{3}y}{\sqrt{2}} - \sqrt{8}y = 0$$

$$\frac{-3y}{\sqrt{2}} - 2\sqrt{2}y = 0$$

$$-3y - 2y = 0$$

$$-5y = 0$$

$$y = \frac{0}{-5}$$

$$\boxed{y = 0}$$

Putting value of y in eqⁿ (i)

$$\sqrt{2}x + 3 \times 0 = 0$$

$$\sqrt{2}x = 0$$

$$x = \frac{0}{\sqrt{2}}$$

$$\boxed{x = 0}$$

(vi) $\frac{3x}{2} - \frac{5y}{3} = -2$; $\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

Sol:: In equation (i)

$$\frac{3x}{2} - \frac{5y}{3} = -2$$

$$\frac{3x}{2} = -2 + \frac{5y}{3}$$

$$\frac{3x}{2} = \frac{-6+5y}{3}$$

$$\cancel{3}x = \frac{2(-6+5y)}{3\times 3}$$

$$x = \frac{-12+10y}{9}$$

Putting value of x in eqn (ii)

$$\frac{-12+10y}{9} \times \frac{1}{3} + \frac{y}{2} = \frac{13}{6}$$

$$\frac{-12+10y}{27} + \frac{y}{2} = \frac{13}{6}$$

$$\frac{-24+20y+27y}{54} = \frac{13}{6}$$

$$-24+47y = \frac{13\times \cancel{54}^{9}}{\cancel{6}_{1}}$$

$$-24+47y = 117$$
$$47y = 117+24$$
$$47y = 141$$

$$y = \frac{\cancel{141}^{3}}{\cancel{47}_{1}}$$

$$\boxed{y = 3}$$

Putting value of y in eqⁿ (i)

$$\frac{3x}{2} - \frac{5}{3} \times 3 = -2$$

$$\frac{3x}{2} = -2 + \frac{15}{3}$$

$$\frac{3x}{2} = \frac{-6+15}{3}$$

$$\frac{3x}{2} = \frac{9}{3}$$

$$x = \frac{\cancel{9}^{\cancel{3}\,1} \times 2}{\cancel{3} \times \cancel{3}}$$

$$\boxed{x = 2}$$

2. Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$

Sol:- In equation (i)

$$2x + 3y = 11$$

$$2x = 11 - 3y$$

$$x = \frac{11 - 3y}{2}$$

Putting value of x in eq^n (ii)

$$2\left(\frac{11-3y}{2}\right) - 4y = -24$$

$$\frac{22-6y}{2} - 4y = -24$$

$$22-6y-8y = -24 \times 2$$
$$22-14y = -48$$
$$-14y = -48-22$$
$$+14y = +70$$
$$y = \frac{70}{14}$$

$$\boxed{y = 5}$$

Putting value of y in eq^n (i)

$$2x + 3 \times 5 = 11$$
$$2x + 15 = 11$$
$$2x = 11-15$$
$$2x = -4$$
$$x = \frac{-4}{2}$$

$$\boxed{x = -2}$$

Now,

$$y = mx + 3$$
$$5 = m \times -2 + 3$$
$$5 - 3 = -2m$$
$$2 = -2m$$

$\frac{\cancel{2}^1}{-\cancel{2}_1} = m$

$\boxed{m = -1}$

3. Form the pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

Ans: The the numbers be 'x' and 'y' where $x > y$

In Ist Case: $x - y = 26$

$x = y + 26$ —— (i)

In IInd Case: $x = 3y$

Putting value of x in eq^n (ii)

$y + 26 = 3y$

$y - 3y = -26$

$+2y = +26$

$y = \frac{26}{2}$

$\boxed{y = 13}$

Putting value of y in eq^n (i)

$x = 13 + 26$

$\boxed{x = 39}$

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

Sol:- Let the numbers be 'x' and 'y' where x > y

In 1st case: $x + y = 180$

$x = 180 - y$ —— (i)

In IInd case: $x = y + 18$

Putting value of x in eqn (ii)

$$180 - y = y + 18$$
$$180 - 18 = y + y$$
$$162 = 2y$$

$$y = \frac{162}{2}$$

$$\boxed{y = 81}$$

Putting value of y in eqn (i)

$$x = 180 - 81$$

$$\boxed{x = 99}$$

(iii) The coach of a cricket team buys 7 bats and 6 balls for ₹3800. Later, she buys 3 bats and 5 balls for ₹1750. Find the cost of each bat and ball.

Sol: Let the cost of bat be 'x' and cost of balls be 'y'

In I^st case: $7x + 6y = 3800$ —— (i)

$$7x = 3800 - 6y$$

$$x = \frac{3800 - 6y}{7}$$ —— (i)

In II^nd case: $3x + 5y = 1750$ —— (ii)

Putting $x = \frac{3800-6y}{7}$ in eq^n (ii)

$$3\left(\frac{3800-6y}{7}\right) + 5y = 1750$$

$$\frac{11400 - 18y}{7} + 5y = 1750$$

$$\frac{11400 - 18y + 35y}{7} = 1750$$

$$11400 + 17y = 1750 \times 7$$

$$11400 + 17y = 12250$$

$$17y = 12250 - 11400$$

$$17y = 850$$

$$y = \frac{850}{17}$$

$$\boxed{y = 50}$$

Putting value of y in eq^n (i)

$7x + 6 \times 50 = 3800$

$7x + 300 = 3800$

$7x = 3800 - 300$

$7x = 3500$

$x = \frac{3500}{7}$

$\boxed{x = 500}$

Hence,

Cost of each bat = ₹500

Cost of each ball = ₹50

(iv) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Sol: Let Jacob's present age be 'x' and his son's age be 'y'

Now,

5 years later,

Jacob's age = $x + 5$

Jabob's son's age = $y + 5$

And,

$x + 5 = 3(y + 5)$

$x + 5 = 3y + 15$

$x = 3y + 15 - 5$

$x = 3y + 10$ —— (i)

Now again,

5 years ago,

Jacob's age = $x - 5$

His son's age = $y - 5$

And,

$$x - 5 = 7(y - 5)$$
$$x - 5 = 7y - 35$$
$$x = 7y - 35 + 5$$
$$x = 7y - 30 \quad \text{—— (ii)}$$

Putting $x = 3y + 10$ in eqⁿ (ii)

$$3y + 10 = 7y - 30$$
$$3y - 7y = -30 - 10$$
$$+4y = +40$$
$$y = \frac{40}{4}$$

$$\boxed{y = 10}$$

Putting volue of y in eqⁿ (i)

$$x = 3 \times 10 + 10$$
$$x = 30 + 10$$
$$\boxed{x = 40}$$

Hence,

Jacob's age = 40 years

His son's age = 10 years

Exersice : 3.4

1. Solve the following pair of linear equations by the elimination method and the substitution method :

(i) $x+y=5$ and $2x-3y=4$

Sol:- **By Elimination Method :-**

$x+y=5$ —— (i)
$2x-3y=4$ —— (ii)

Multiply by 2 in eq^n (i)

$2x+2y=10$ —— (iii)

Subtract eq^n (ii) from (iii)

$$\begin{array}{r} 2x+2y=10 \\ 2x-3y=4 \\ -\quad+\quad- \\ \hline 5y=6 \end{array}$$

$$\boxed{y=\frac{6}{5}}$$

Putting value of y in eq^n (i)

$$x+\frac{6}{5}=5$$

$$x=5-\frac{6}{5}$$

$$x = \frac{25-6}{5}$$

$$\boxed{x = \frac{19}{5}}$$

By Substitution Method :-

$x + y = 5$ —— (i)

$x = 5 - y$ —— (i)

$2x - 3y = 4$ —— (ii)

Putting $x = 5-y$ in eq^n (ii)

$$2(5-y) - 3y = 4$$
$$10 - 2y - 3y = 4$$
$$-5y = 4 - 10$$
$$+5y = +6$$
$$\boxed{y = \frac{6}{5}}$$

Putting value of y in eq^n (i)

$$x + \frac{6}{5} = 5$$
$$x = 5 - \frac{6}{5}$$
$$x = \frac{25-6}{5}$$
$$\boxed{x = \frac{19}{5}}$$

(ii) $3x+4y=10$ and $2x-2y=2$

Sol:- By Elimination Method :

$3x+4y=10$ ——— (i)
$2x-2y=2$ ——— (ii)

Multiply by 2 in eq^n (ii)

$4x-4y=4$ ——— (iii)

~~Subtract~~ eq^n (i) from (iii)

$$\begin{aligned} 4x-4y &= 4 \\ 3x+4y &= 10 \\ \hline 7x &= 14 \end{aligned}$$

$$x = \frac{14}{7}$$

$$\boxed{x=2}$$

Putting value of x in eq^n (i)

$$3\times2+4y = 10$$
$$4y = 10-6$$
$$4y = 4$$
$$y = \frac{4}{4}$$

$$\boxed{y=1}$$

By Substitution Method :

$3x + 4y = 10$ ——— (i)

$3x = 10 - 4y$

$x = \frac{10 - 4y}{3}$

$2x - 2y = 2$ ——— (ii)

Putting ~~val~~ $x = \frac{10 - 4y}{3}$ in eq^n (ii)

$2\left(\frac{10 - 4y}{3}\right) - 2y = 2$

$\frac{20 - 8y}{3} - 2y = 2$

$\frac{20 - 8y - 6y}{3} = 2$

$20 - 14y = 2 \times 3$

$-14y = 6 - 20$

$+14y = +14$

$y = \frac{14}{14}$

$\boxed{y = 1}$

Putting value of y in eq^n (i)

$3x + 4 \times 1 = 10$

$3x = 10 - 4$

$3x = 6$

$x = \frac{6}{3}$

$\boxed{x = 2}$

(iii) $3x - 5y - 4 = 0$ and $9x = 2y + 7$

Sol:- $3x - 5y = 4$; $9x - 2y = 7$

By Elimination Method :

$3x - 5y = 4$ —— (i)

$9x - 2y = 7$ —— (ii)

Multiply by 3 in eqⁿ (i)

$3(3x - 5y) = 3 \times 4$

$9x - 15y = 12$ —— (iii)

Subtract eqⁿ (ii) from (iii)

$$\begin{array}{r} 9x - 15y = 12 \\ 9x - 2y = 7 \\ - \quad + \quad\quad - \\ \hline -13y = 5 \end{array}$$

$$\boxed{y = \frac{-5}{13}}$$

Putting value of y in eqⁿ (i)

$3x - 5 \times \frac{-5}{13} = 4$

$3x + \frac{25}{13} = 4$

$$3x = 4 - \frac{25}{13}$$

$$3x = \frac{52-25}{13}$$

$$x = \frac{\cancel{27}^{9}}{13} \times \frac{1}{\cancel{3}_1}$$

$$\boxed{x = \frac{9}{13}}$$

By Substitution Method :

$$3x - 5y = 4 \quad \text{——— (i)}$$

$$3x = 4 + 5y$$

$$x = \frac{4+5y}{3}$$

$$9x - 2y = 7 \quad \text{——— (ii)}$$

Putting $x = \frac{4+5y}{3}$ in eq^n (ii)

$$9\left(\frac{4+5y}{3}\right) - 2y = 7$$

$$\frac{36+45y}{3} - 2y = 7$$

$$\frac{36+45y-6y}{3} = 7$$

$36 + 39y = 21$

$39y = 21 - 36$

$39y = -15$

$y = -\frac{15}{39}$

$\boxed{y = \frac{-5}{13}}$

Putting value of y in eq^n (i)

$3x - 5\left(\frac{-5}{13}\right) = 4$

$3x + \frac{25}{13} = 4$

$3x = 4 - \frac{25}{13}$

$3x = \frac{52 - 25}{13}$

$x = \frac{\cancel{27}^{9}}{13} \times \frac{1}{\cancel{3}_{1}}$

$\boxed{x = \frac{9}{13}}$

(iv) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$

Sol: By Elimination Method :

$$\frac{x}{2} + \frac{2y}{3} = -1$$

Multiply by 6

$$\frac{x}{\cancel{2}_1} \times \cancel{6}^3 + \frac{2y}{\cancel{3}_1} \times \cancel{6}^2 = -1 \times 6$$

$$3x + 4y = -6 \quad \text{——— (i)}$$

$$x - \frac{y}{3} = 3$$

Multiply by 3

$$3x - \frac{y}{3} \times 3 = 3 \times 3$$

$$3x - y = 9 \quad \text{——— (ii)}$$

Subtract eqn (ii) from (i)

$$\begin{array}{rcr} 3x + 4y & = & -6 \\ 3x - y & = & 9 \\ - \quad + & & - \\ \hline 5y & = & -15 \end{array}$$

$$y = \frac{-15}{5}$$

$$\boxed{y = -3}$$

Putting value of y in eq^n (i)

$$3x + 4(-3) = -6$$
$$3x - 12 = -6$$
$$3x = -6 + 12$$
$$3x = 6$$
$$x = \frac{6}{3}$$

$$\boxed{x = 2}$$

By Substitution Method:

$$\frac{x}{2} + \frac{2y}{3} = -1$$

$$\frac{3x + 4y}{6} = -1$$

$$3x + 4y = -6$$
$$3x = -6 - 4y$$
$$x = \frac{-6 - 4y}{3} \text{ ——— (i)}$$

$$x - \frac{y}{3} = 3$$

$$\frac{3x - y}{3} = 3$$

$$3x - y = 9 \text{ ——— (ii)}$$

Putting $x = \frac{-6 - 4y}{3}$ in eq^n (ii)

$$x\left(\frac{-6+4y}{x}\right) - y = 9$$

$$-6+4y - y = 9$$
$$-5y = 9+6$$
$$y = \frac{-15}{5}$$

$\boxed{\cancel{y = 5}}$ $\boxed{y = -3}$

Putting value of y in eq^n (ii)

$$3x - (-3) = 9$$
$$3x + 3 = 9$$
$$3x = 9 - 3$$
$$x = \frac{6}{3}$$

$\boxed{x = 2}$

2. From the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method :

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction ?

Sol:- Let the numerator be 'x' and denominator by 'y'
Then,

$$\text{Fraction} = \frac{x}{y}$$

Condition I : $\frac{x+1}{y-1} = 1$

$$x+1 = y-1$$
$$x-y = -2 \quad \text{——— (i)}$$

Condition II : $\frac{x}{y+1} = \frac{1}{2}$

$$2x = y+1$$
$$2x-y = 1 \quad \text{——— (ii)}$$

Subtracting ~~Adding~~ eq^n (i) from (ii)

$$\begin{array}{r} 2x - y = 1 \\ x - y = -2 \\ - \quad + \quad\quad + \\ \hline x = 3 \end{array}$$

$$\boxed{x = 3}$$

Putting value of x in eq^n (i)

$$3 - y = -2$$

$$-y = -2-3$$
$$+y = +5$$

$$\boxed{y = 5}$$

Hence,

Fraction = $\frac{3}{5}$

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

Sol:- Let Nuri's present age be 'x'
& Sonu's present age be 'y'

5 years ago;

Nuri's age $= x - 5$
Sonu's age $= y - 5$

Then,

$$x - 5 = 3(y - 5)$$
$$x - 5 = 3y - 15$$
$$x - 3y = -10 \quad \text{——— (i)}$$

And,

10 years later;

Nuri's age $= x + 10$
Sonu's age $= y + 10$

Then,

$$x + 10 = 2(y + 10)$$
$$x + 10 = 2y + 20$$
$$x - 2y = 20 - 10$$
$$x - 2y = 10 \quad \text{——— (ii)}$$

Subtract eqn (i) from (ii)

$$x - 2y = 10$$
$$x - 3y = -10$$
$$- \quad + \qquad +$$
$$\overline{\qquad y = 20 \qquad}$$

$$\boxed{y = 20}$$

Putting value of y in eq^n (ii)

$$x - 2 \times 20 = 10$$

$$x - 40 = 10$$

$$x = 10 + 40$$

$$\boxed{x = 50}$$

Hence,

Nuri's age = 50 years

Sonu's age = 20 years

(iv) Meena went to a bank to withdraw ₹2000. She asked the cashier to give her ₹50 and ₹100 notes only. Meena got 25 notes in all. Find how many notes of ₹50 and ₹100 she recieved.

Sol:- Let no. of ₹50 notes be 'x'
& no. of ₹100 notes be 'y'

Now,

Condition I: $50x + 100y = 2000$

$$50(x + 2y) = 2000$$

$$x + 2y = \frac{\cancel{2000}\ 40}{\cancel{50}\ 1}$$

$$x + 2y = 40 \quad \text{——— (i)}$$

Condition II : $x + y = 25$ —— (ii)

~~Mu~~ Subtract eq^n (ii) from (i)

$$x + 2y = 40$$
$$x + y = 25$$
$$- \quad - \quad -$$
$$y = 15$$

$$y = \frac{15}{}$$

$$\boxed{y = 15}$$

Putting value of y in eq^n (ii)

$$x + 15 = 25$$
$$x = 25 - 15$$
$$\boxed{x = 10}$$

Hence,
No. of ₹50 notes = 10
No. of ₹100 notes = 15

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹27 for a book kept for seven days, while Susy paid ₹21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Sol:- Let the fixed charge be 'x'
and charge for each extra day be 'y'

Condition I: $x + 4y = 27$ —— (i)

Condition II: $x + 2y = 21$ —— (ii)

Subtract eqn (ii) from (i)

$$x + 4y = 27$$
$$x + 2y = 21$$
$$- \quad - \quad -$$

$$2y = 6$$

$$y = \frac{6}{2}$$

$$\boxed{y = 3}$$

Putting value of y in eqn (ii)

$$x + 2 \times 3 = 21$$
$$x + 6 = 21$$
$$x = 21 - 6$$
$$\boxed{x = 15}$$

Hence,

Fixed charge = ₹15

Charge for each extra day = ₹3

Exercise : 3.5

1. Which of the following pairs of linear equation has unique solution, no solution, or infinitely many solutions. In case there is a unique solution, find it by using cross multiplication method:

(i) $x - 3y - 3 = 0$; $3x - 9y - 2 = 0$

Sol: Comparing both equations, we get

$$\frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{\cancel{+3}}{\cancel{+9}_3} = \frac{1}{3}, \quad \frac{c_1}{c_2} = \frac{+3}{+2}$$

Here,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence,

Linear equations have no solution.

(ii) $2x + y = 5$; $3x + 2y = 8$

Sol: $2x + y - 5 = 0$; $3x + 2y - 8 = 0$

Comparing both equations, we get

$$\frac{a_1}{a_2} = \frac{2}{3}, \quad \frac{b_1}{b_2} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{\cancel{+5}}{\cancel{+8}} \; \frac{-5}{-8}$$

Here,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Hence,

Linear equations have a unique solution

By Cross Multiplication Method :

$$\begin{array}{cccccccc} & x & & & y & & & 1 \\ 1 & & -5 & & 2 & & 1 & \\ 2 & & -8 & & 3 & & 2 & \end{array}$$

$$\frac{x}{1\times(-8)-2\times(-5)} = \frac{y}{3\times(-5)-2\times(-8)} = \frac{1}{2\times2-3\times1}$$

$$\frac{x}{-8+10} = \frac{y}{-15+16} = \frac{1}{4-3}$$

$$\frac{x}{2} = \frac{y}{1} = \frac{1}{1}$$

Now,

$$\frac{x}{2} = \frac{1}{1}$$

$$x\times1 = 2\times1$$

$$\boxed{x = 2}$$

$$\frac{y}{1} = \frac{1}{1}$$

$$y\times1 = 1\times1$$

$$\boxed{y = 1}$$

(iii) $3x - 5y = 20$; $6x - 10y = 40$

Sol:- $3x - 5y - 20 = 0$; $6x - 10y - 40 = 0$

Comparing both equations, we get

$\frac{a_1}{a_2} = \frac{\cancel{3}^1}{\cancel{6}_2} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{\cancel{-5}^1}{\cancel{-10}_2} = \frac{1}{2}$, $\frac{c_1}{c_2} = \frac{\cancel{-20}^1}{\cancel{-40}_2} = \frac{1}{2}$

Here,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence,

It has infinitely many solution.

(iv) $x - 3y - 7 = 0$; $3x - 3y - 15 = 0$

Sol:. Comparing both equations, we get

$\frac{a_1}{a_2} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{-3}{-3}$, $\frac{c_1}{c_2} = \frac{-7}{-15}$

Here,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Hence,

It has a unique solution.

By Cross Multiplication Method :

x y 1

$\frac{-3}{-3}$ $\frac{-7}{-15}$ $\frac{1}{3}$ $\frac{-3}{-3}$

$$\frac{x}{(-3)\times(-15) - (-3)\times(-7)} = \frac{y}{(-7)\times 3 - (-15)\times 1} = \frac{1}{1\times(-3) - (3\times -3)}$$

$$\frac{x}{45-21} = \frac{y}{-21-(-15)} = \frac{1}{-3+9}$$

$$\frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$$

Now,

$$\frac{x}{24} = \frac{1}{6}$$

$$x \times 6 = 24 \times 1$$

$$x = \frac{24}{6}$$

$$\boxed{x = 4}$$

$$\frac{y}{-6} = \frac{1}{6}$$

$$y \times 6 = 1 \times (-6)$$

$$y = \frac{-6}{6}$$

$$\boxed{y = -1}$$

4 Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method :

(i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay ₹1000 as hostel charges whereas a student B, who takes food for 26 days, pays ₹1180 as hostel charges. Find the fixed charges and the cost of food per day

Sol:- Let the fixed charges be 'x' and cost of food per day be 'y'

Condition I : $x + 20y = ₹1000$ —— (i)

Condition II : $x + 26y = 1180$ —— (ii)

Subtract eqⁿ (i) from (ii)

$$\begin{array}{r} x + 26y = 1180 \\ x + 20y = 1000 \\ - \quad - \quad\quad - \\ \hline 6y = 180 \end{array}$$

$$y = \frac{180}{6}$$

$$\boxed{y = 30}$$

Putting value of y in eq^n (i)

$$x + 20 \times 30 = 1000$$
$$x + 600 = 1000$$
$$x = 1000 - 600$$

$$\boxed{x = 400}$$

Hence,
Fixed charge = ₹400
Cost of food per day = ₹30

(ii) A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

Sol:- Let the numerator be 'x'
& the denominator be 'y'
Then,
Fraction $= \frac{x}{y}$

Condition I: $\frac{x-1}{y} = \frac{1}{3}$

$$3(x-1) = y$$
$$3x - 3 = y$$
$$3x - y = 3 \quad \text{—— (i)}$$

Condition II: $\frac{x}{y+8} = \frac{1}{4}$

$$4x = y + 8$$
$$4x - y = 8 \quad \text{—— (ii)}$$

Subtract eqⁿ (i) from (ii)

$$\begin{array}{r} 4x - y = 8 \\ 3x - y = 3 \\ - \quad + \quad\quad - \\ \hline x = 5 \end{array}$$

$\boxed{x = 5}$

Putting value of x in eqⁿ (ii)

$4 \times 5 - y = 8$

$-y = 8 - 20$

$+y = +12$

$\boxed{y = 12}$

Fraction $= \frac{5}{12}$

(iii) Yash scored 40 marks in a test getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

Sol:- Let no. of correct answers be 'x'
& no. of incorrect answers be 'y'

Condition I: $3x - y = 40$ —— (i)

Condition II : $4x - 2y = 50$ —— (ii)

Multiply by 2 in eqⁿ (i)

$6x - 2y = 80$ —— (iii)

Subtract eqⁿ (ii) from (iii)

$$\begin{array}{r} 6x - 2y = 80 \\ 4x - 2y = 50 \\ - \quad + \quad - \\ \hline 2x = 30 \end{array}$$

$$x = \frac{30}{2}$$

$$\boxed{x = 15}$$

Putting value of x in eqⁿ (i)

$$3 \times 15 - y = 40$$
$$45 - y = 40$$
$$-y = 40 - 45$$
$$\cancel{-}y = \cancel{-}5$$

$$\boxed{y = 5}$$

Hence,

No. of correct answers = 15

No. of incorrect answer = 5

∴ Total questions = 15 + 5 = 20

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

Sol:- Let the speed of cars be 'x & y'
And,

Distance of A from highway = 100 km
Distance of B from highway = 100 km

Condition I : $x + y = 100$ —— (i)

Condition II : $x - y = \frac{100}{5}$

$x - y = 20$ —— (ii)

Subtract eq^n (ii) from (i)

$$\begin{array}{r} x + y = 100 \\ x - y = 20 \\ - \quad + \quad\quad - \\ \hline 2y = 80 \end{array}$$

$y = \frac{80}{2}$

$\boxed{y = 40}$

Putting value of y in eq^n (ii)

$$x - 40 = 20$$
$$x = 20 + 40$$

$$\boxed{x = 60}$$

(v) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Sol:- Let the length of rectangle be 'x'
& the breadth of rectangle be 'y'
Then,
Area of rectangle = xy

Condition I : $(x - 5)(y + 3) = xy - 9$
$$xy + 3x - 5y - 15 = xy - 9$$
$$xy + 3x - 5y - xy = -9 + 15$$
$$3x - 5y = 6 \quad \text{—— (i)}$$

Condition II : $(x + 3)(y + 2) = xy + 67$
$$xy + 2x + 3y + 6 = xy + 67$$
$$xy + 2x + 3y - xy = 67 - 6$$
$$2x + 3y = 61 \quad \text{—— (ii)}$$

Now,
Multiply by 2 in eqⁿ (i)

$$6x - 10y = 12 \quad \text{—— (iii)}$$

Multiply by 3 in eqⁿ (ii)

$$6x + 9y = 183 \quad \text{—— (iv)}$$

Subtract eq^n (iii) from (iv)

$$\begin{array}{r} 6x + 9y = 183 \\ 6x - 10y = 12 \\ - \quad + \quad\quad - \\ \hline 19y = 171 \end{array}$$

$$y = \frac{171}{19}$$

$$\boxed{y = 9}$$

Putting value of y in eq^n (i)

$$3x - 5 \times 9 = 6$$

$$3x = 6 + 45$$

$$x = \frac{51}{3}$$

$$\boxed{x = 17}$$

Hence,

Length = 17 unit

Breadth = 9 unit.

Exercise - 3.6

1. Solve the following pairs of linear equations by reducing them to a pair of linear equations:

(i) $\frac{1}{2x} + \frac{1}{3y} = 2$; $\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$

Sol:- Let $\frac{1}{x} = a$ & $\frac{1}{y} = b$

Then,

$$\frac{a}{2} + \frac{b}{3} = 2$$

$$\frac{3a+2b}{6} = 2$$

$$3a + 2b = 12 \quad \text{——— (i)}$$

And,

$$\frac{a}{3} + \frac{b}{2} = \frac{13}{6}$$

$$\frac{2a+3b}{6} = \frac{13}{6}$$

$$2a + 3b = \frac{13}{6} \times 6$$

$$2a + 3b = 13 \quad \text{——— (ii)}$$

In eq^n (i)

$$3a + 2b = 12$$

$$3a = 12 - 2b$$

$$a = \frac{12-2b}{3} \quad \text{——— (iii)}$$

Put $a = \frac{12-2b}{3}$ in eq^n (ii)

$$2a + 3b = 13$$

$$2\left(\frac{12-2b}{3}\right) + 3b = 13$$

$$\frac{24-4b}{3} + 3b = 13$$

$$24 - 4b + 9b = 13 \times 3$$

$$5b = 39 - 24$$

$$b = \frac{15}{5}$$

$$b = 3$$

Put $b = 3$ in eq^n (iii)

$$a = \frac{12-2b}{3}$$

$$a = \frac{12-2\times 3}{3}$$

$$a = \frac{12-6}{3}$$

$$a = \frac{6}{3}$$

$$a = 2$$

Now,

$$a = 2$$

$$\frac{1}{x} = 2 \qquad \left[\because a = \frac{1}{x}\right]$$

$$\boxed{x = \frac{1}{2}}$$

And,

$$b = 3$$

$$\frac{1}{y} = 3 \qquad \left[\because b = \frac{1}{y}\right]$$

$\boxed{y = \frac{1}{3}}$

(ii) $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$; $\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$

Sol:- Let $\frac{1}{\sqrt{x}} = a$ & $\frac{1}{\sqrt{y}} = b$

Then,

$2a + 3b = 2$ —— (i)

$4a - 9b = -1$ —— (ii)

Multiply by 3 in eq^n (i)

$6a + 9b = 6$ —— (iii)

Adding eq^n (ii) & (iii)

$$\begin{array}{r} 6a + 9b = 6 \\ 4a - 9b = -1 \\ \hline 10a \quad = 5 \end{array}$$

$a = \frac{5}{10}$

$a = 1/2$

Put $a = \frac{1}{2}$ in eq^n (i)

$2a + 3b = 2$

$2 \times \frac{1}{2} + 3b = 2$

$3b = 2 - 1$

$b = 1/3$

$a = \frac{1}{2}$

$\frac{1}{\sqrt{x}} = \frac{1}{2}$

$\sqrt{x} = 2$

Squaring both sides

$(\sqrt{x})^2 = (2)^2$

$x = 4$

$b = \frac{1}{3}$

$\frac{1}{\sqrt{y}} = \frac{1}{3}$

$\sqrt{y} = 3$

Squaring both sides

$(\sqrt{y})^2 = (3)^3$

$y = 9$

(iii) $\frac{4}{x} + 3y = 14$; $\frac{3}{x} - 4y = 23$

Sol:- Let $\frac{1}{x} = a$

Then,

$4a + 3y = 14$ — (i); $3a - 4y = 23$ — (ii)

In eq^n (ii)

$3a - 4y = 23$

$3a = 23 + 4y$

$a = \frac{23 + 4y}{3}$ — (iii)

Put $a = \frac{23 - 4y}{3}$ in eq^n (i)

$4\left(\frac{23 + 4y}{3}\right) + 3y = 14$

$\frac{92 + 16y}{3} + 3y = 14$

$92 + 16y + 9y = 14 \times 3$

$92 + 25y = 42$

$-25y = 42 - 92$

$-25y = 50$

$y = \frac{-50}{25}$

$\boxed{y = -2}$

Putting $y = -2$ in eq^n (iii)

$a = \frac{23 + 4y}{3}$

$a = \frac{23 + 4 \times -2}{3}$

$a = \frac{23 - 8}{3}$

$a = \frac{15}{3}$

$a = 5$

$\frac{1}{x} = 5$

$\boxed{x = \frac{1}{5}}$

(iv) $\frac{5}{x-1} + \frac{1}{y-2} = 2$; $\frac{6}{x-1} - \frac{3}{y-2} = 1$

Sol:- Let $\frac{1}{x-1} = a$ & $\frac{1}{y-2} = b$

$5a + b = 2$ ——— (i)

$6a - 3b = 1$ ——— (ii)

Multiply by 3 in eq^n (i)

$15a + 3b = 6$ —— (iii)

Adding eq^n (ii) and (iii)

$$\begin{array}{r} 15a + 3b = 6 \\ 6a - 3b = 1 \\ \hline 21a \quad = 7 \end{array}$$

$$a = \frac{7}{21}$$

$$a = \frac{1}{3}$$

Putting $a = 1/3$ in eq^n (ii)

$$6a - 3b = 1$$

$$\cancel{6}^{2} \times \frac{1}{\cancel{3}_{1}} - 3b = 1$$

$$2 - 3b = 1$$

$$-3b = 1 - 2$$

$$+3b = +1$$

$$b = \frac{1}{3}$$

$a = \frac{1}{3}$	$b = \frac{1}{3}$
$\frac{1}{x-1} = \frac{1}{3}$	$\frac{1}{y-2} = \frac{1}{3}$
$x - 1 = 3$	$y - 2 = 3$
$x = 3 + 1$	$y = 3 + 2$
$x = 4$	$y = 5$

(v) $\frac{7x - 2y}{xy} = 5$; $\frac{8x + 7y}{xy} = 15$

Sol:- $\frac{7x}{xy} - \frac{2y}{xy} = 5$; $\frac{8x}{xy} + \frac{7y}{xy} = 15$

$\frac{7}{y} - \frac{2}{x} = 5$; $\frac{8}{y} + \frac{7}{x} = 15$

Let $\frac{1}{y} = a$ and $\frac{1}{x} = b$

$7a - 2b = 5$ ——— (i)

$8a + 7b = 15$

$8a = 15 - 7b$

$a = \frac{15 - 7b}{8}$ ——— (ii)

Putting $a = \frac{15 - 7b}{8}$ in eqⁿ (i)

$7a - 2b = 5$

$7\left(\frac{15 - 7b}{8}\right) - 2b = 5$

$\frac{105 - 49b}{8} - 2b = 5$

$105 - 49b - 16b = 5 \times 8$

$-65b = 40 - 105$

$+65b = +65$

$b = \frac{65}{65}$

$b = 1$

Putting $b = 1$ in eq^n (ii)

$a = \frac{15 - 7b}{8}$

$a = \frac{15 - 7 \times 1}{8}$

$a = \frac{15 - 7}{8}$

$a = \frac{8}{8}$

$a = 1$

$a = 1$	$b = 1$
$\frac{1}{y} = 1$	$\frac{1}{x} = 1$
$\boxed{y = 1}$	$\boxed{x = 1}$

(vi) $6x + 3y = 6xy \quad ; \quad 2x + 4y = 5xy$

Sol: Divide by xy on both sides in both equations:

$\frac{6x}{xy} + \frac{3y}{xy} = \frac{6xy}{xy} \quad ; \quad \frac{2x}{xy} + \frac{4y}{xy} = \frac{5xy}{xy}$

$\frac{6}{y} + \frac{3}{x} = 6 \quad ; \quad \frac{2}{y} + \frac{4}{x} = 5$

Let $\frac{1}{y} = a$ & $\frac{1}{x} = b$

$6a + 3b = 6$ —— (i)

$2a + 4b = 5$

$2a = 5 - 4b$

$a = \frac{5-4b}{2}$ —— (ii)

Putting $a = \frac{5-4b}{2}$ in eqⁿ (i)

$6a + 3b = 6$

$\cancel{6}^{3} \times \frac{5-4b}{\cancel{2}} + 3b = 6$

$15 - 12b + 3b = 6$

$-9b = 6 - 15$

$+9b = +9$

$b = \frac{9}{9}$

$b = 1$

Put $b = 1$ in eqⁿ (ii)

$a = \frac{5-4b}{2}$

$a = \frac{5-4\times 1}{2}$

$a = \frac{5-4}{2}$

$a = \frac{1}{2}$

Now,

$a = \frac{1}{2}$

$\frac{1}{y} = \frac{1}{2}$

$\boxed{y = 2}$

$b = 1$

$\frac{1}{x} = 1$

$\boxed{x = 1}$

(vii) $\frac{10}{x+y} + \frac{2}{x-y} = 4$

$\frac{15}{x+y} - \frac{5}{x-y} = -2$

Sol: Let $\frac{1}{x+y} = a$ & $\frac{1}{x-y} = b$

$10a + 2b = 4$ —— (i)

$15a - 5b = -2$ —— (ii)

In equation (i)

$10a + 2b = 4$

$10a = 4 - 2b$

$a = \frac{4-2b}{10}$ —— (iii)

Putting $a = \frac{4-2b}{10}$ in eq^n (ii)

$$15a - 5b = -2$$

$$15 \times \frac{4-2b}{10} - 5b = -2$$

$$\cancel{15}^{3} \times \frac{\cancel{2}^{1}(2-b)}{\cancel{10}\,\cancel{5}_{1}} - 5b = -2$$

$$6 - 3b - 8b = -2$$

$$-8b = -2 - 6$$

$$+8b = +8$$

$$b = \frac{8}{8}$$

$$b = 1$$

Putting $b = 1$ in eqn (iii)

$$a = \frac{4-2b}{10}$$

$$a = \frac{4-2\times 1}{10}$$

$$a = \frac{4-2}{10}$$

$$a = \frac{2}{10}$$

$$a = \frac{1}{5}$$

Now,

$$a = \frac{1}{5} \qquad b = 1$$

$$\frac{1}{x+y} = \frac{1}{5} \qquad \frac{1}{x-y} = 1$$

$$x + y = 5 \text{ —— (iv)} \qquad x - y = 1 \text{ —— (v)}$$

Adding eqⁿ (iv) & (v)

$$x + y = 5$$
$$x - y = 1$$
$$2x = 6$$
$$x = \frac{6}{2}$$

$$\boxed{x = 3}$$

Putting $x = 3$ in eqⁿ (iv)

$$x + y = 5$$
$$3 + y = 5$$
$$y = 5 - 3$$

$$\boxed{y = 2}$$

(viii) $\dfrac{1}{3x+y} + \dfrac{1}{3x-y} = \dfrac{3}{4}$

$\dfrac{1}{2(3x+y)} - \dfrac{1}{2(3x-y)} = \dfrac{-1}{8}$

Sol:- Let $\dfrac{1}{3x+y} = a$ & $\dfrac{1}{3x-y} = b$

$a + b = \dfrac{3}{4}$ ——

$4a + 4b = 3$ —— (i)

$\dfrac{a}{2} - \dfrac{b}{2} = \dfrac{-1}{8}$

$$\frac{a-b}{2} = \frac{-1}{8}$$

$$a-b = \frac{-1 \times \cancel{2}^{1}}{\cancel{8}_{4}}$$

$$a-b = \frac{-1}{4}$$

$$4a - 4b = -1 \quad \text{—— (ii)}$$

Adding eq^n (i) and (ii)

$$\begin{array}{l} 4a + 4b = 3 \\ 4a - 4b = -1 \\ \hline 8a \quad\quad = 2 \end{array}$$

$$a = \frac{2}{8}$$

$$a = \frac{1}{4}$$

Putting $a = \frac{1}{4}$ in eq^n (i)

$$a + b = \frac{3}{4}$$

$$\frac{1}{4} + b = \frac{3}{4}$$

$$b = \frac{3}{4} - \frac{1}{4}$$

$$b = \frac{3-1}{4}$$

$$b = \frac{2}{4}$$

$b = \frac{1}{2}$

Now,

$a = \frac{1}{4}$ $b = \frac{1}{2}$

$\frac{1}{3x+y} = \frac{1}{4}$ $\frac{1}{3x-y} = \frac{1}{2}$

$3x + y = 4$ —— (iii) $3x - y = 2$ —— (iv)

Adding eq^n (iii) & (iv)

$$3x + y = 4$$
$$3x - y = 2$$

$$6x = 6$$

$$x = \frac{6}{6}$$

$$\boxed{x = 1}$$

Put $x = 1$ in eq^n (iii)

$$3x + y = 4$$
$$3 \times 1 + y = 4$$
$$y = 4 - 3$$

$$\boxed{y = 1}$$

2. Formulate the following problems as a pair of equations, and hence find their solutions

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and speed of current.

Sol:- Let the speed of rowing in still water = x
& the speed of current = y

Now,

Speed of rowing in downstream = $\frac{20}{2}$

$$x + y = 10 \quad \text{—— (i)}$$

And,

Speed of rowing in upstream = $\frac{4}{2}$

$$x - y = 2 \quad \text{—— (ii)}$$

Adding eqn (i) & (ii)

$$\begin{aligned} x + y &= 10 \\ x - y &= 2 \\ \hline 2x &= 12 \\ x &= \frac{12}{2} \\ \boxed{x = 6} \end{aligned}$$

Putting value of x in eqn (i)

$$\begin{aligned} x + y &= 10 \\ 6 + y &= 10 \\ y &= 10 - 6 \\ \boxed{y = 4} \end{aligned}$$

Hence,

Speed of rowing in still water = 6 km/hr

Speed of current = 4 km/hr.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 women alone to finish the work, and also that taken by 1 man alone.

Sol:: Given, Time taken by 2 women + 5 man to do work = 4 day

Time taken by 3 women + 6 men to do work = 3 day

Now,

Let the time taken by 1 woman to do work be 'x'

& the time taken by 1 man to do work be 'y'

So,

Work done by 1 woman in 1 day = $\frac{1}{x}$

Work done by 1 man in 1 day = $\frac{1}{y}$

According to condition I:

$$4\left(\frac{2}{x} + \frac{5}{y}\right) = 1$$

$$\frac{8}{x} + \frac{20}{y} = 1$$

Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$

$$8a + 20b = 1 \quad \text{—— (i)}$$

According to Condition II

$$3\left(\frac{3}{x} + \frac{6}{y}\right) = 1$$

$$\frac{9}{x} + \frac{18}{y} = 1$$

$$9a + 18b = 1 \quad \text{——— (ii)}$$

In eq^n (i)

$$8a + 20b = 1$$
$$8a = 1 - 20b$$
$$a = \frac{1-20b}{8} \quad \text{——— (iii)}$$

Putting $a = \frac{1-20b}{8}$ in eq^n (ii)

$$9a + 18b = 1$$
$$9\left(\frac{1-20b}{8}\right) + 18b = 1$$

$$\frac{9-180b}{8} + 18b = 1$$

$$9 - 180b + 144b = 8$$
$$-36b = 8 - 9$$
$$+36b = +1$$
$$b = \frac{1}{36}$$

Putting $b = 1/36$ in eqⁿ (iii)

$$a = \frac{1 - 20b}{8}$$

$$a = \frac{1 - \cancel{20}^{5} \times \frac{1}{\cancel{36}\,9}}{8}$$

$$a = \frac{1 - \frac{5}{9}}{8}$$

$$a = \frac{\frac{9-5}{9}}{8}$$

$$a = \frac{\frac{4}{9}}{8}$$

$$a = \frac{\cancel{4}^{1}}{9} \times \frac{1}{\cancel{8}_{2}}$$

$$a = \frac{1}{18}$$

Now,

$$a = \frac{1}{18} \qquad b = \frac{1}{36}$$

$$\frac{1}{x} = \frac{1}{18} \qquad \frac{1}{y} = \frac{1}{36}$$

$$x = 18 \qquad y = 36$$

Hence,

~~Ta~~ Time taken by 1 woman to finish a work: 18 days

Time taken by 1 man to finish a work: 36 days

(ii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and remaining by bus

If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of train and the bus separately.

Sol:- Given, Total distance = 300 km

Let the speed of the train be 'x'
& the speed of the bus be 'y'

Condition I:

Distance travelled by train = 60 km
Distance travelled by bus = (300 − 60) km = 240 km

Then,

$$\frac{60}{x} + \frac{240}{y} = 4$$

Let $\frac{1}{x} = a$ & $\frac{1}{y} = b$

$$60a + 240b = 4$$

$$60(a + 4b) = 4$$

$$a + 4b = \frac{4}{60}$$

$$a + 4b = \frac{1}{15}$$

$$a = \frac{1}{15} - 4b$$

$$a = \frac{1 - 60b}{15} \quad \text{—— (i)}$$

Condition II:

Distance travelled by train = 100 km
Distance travelled by bus = (300 − 100) km = 200 km

$$\frac{100}{x} + \frac{200}{y} = 4 + \frac{1\cancel{0}}{6\cancel{0}}$$

$$100a + 200b = \frac{24+1}{6}$$

$$100(a+2b) = \frac{25}{6}$$

$$a+2b = \frac{\cancel{25}\,1}{6\times\cancel{100}\,4}$$

$$a+2b = \frac{1}{24} \quad \text{—— (ii)}$$

Putting $a = \frac{1-60b}{15}$ in eq^n (ii)

$$\frac{1-60b}{15} + 2b = \frac{1}{24}$$

$$1 - 60b + 30b = \frac{15}{24}$$

$$-30b = \frac{15}{24} - 1$$

$$-30b = \frac{15-24}{15}$$

$$\cancel{-}30b = \frac{\cancel{-}9}{24}$$

$$b = \frac{\cancel{9}\,3\,1}{\cancel{24}_{8}\times\cancel{30}_{10}}$$

$$b = \frac{1}{80}$$

Put $b = \frac{1}{80}$ in eq^n (i)

$$a = \frac{1 - 60b}{15}$$

$$a = \frac{1 - \cancel{60}^{3} \times \frac{1}{\cancel{80}_{4}}}{15}$$

$$a = \frac{1 - \frac{3}{4}}{15}$$

$$a = \frac{\frac{4-3}{4}}{15}$$

$$a = \frac{\frac{1}{4}}{15}$$

$$a = \frac{1}{4} \times \frac{1}{15}$$

$$a = \frac{1}{60}$$

Now,

$$a = \frac{1}{60} \qquad b = \frac{1}{80}$$

$$\frac{1}{x} = \frac{1}{60} \qquad \frac{1}{y} = \frac{1}{80}$$

$$x = 60 \qquad y = 80$$

Hence,

speed of train = 60 km/hr

Speed of bus = 80 km/hr.

Summary

1. Two linear equations in the same two variables are called a pair of linear equations in two variables. The most general form of a pair of linear equations is

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

where $a_1, a_2, b_1, b_2, c_1, c_2$ are real numbers, such that $a_1^2 + b_1^2 \neq 0$, $a_2^2 + b_2^2 \neq 0$.

2. A pair of linear equations in two variables can be represented, and solved, by the

(i) Graphical method (ii) Algebraic Method

3. If a pair of linear equations is given by $a_1x + b_1y + c = 0$ and $a_2x + b_2y + c_2 = 0$, then the following situation can arise

Compare the ratios	Graphical representation	Algebraic representation
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Exactly one solution (unique)
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution