Chapter - 1

Real Numbers

Euclid's Division Lemma: Given positive integers a and b, there exist unique integers q and r satisfying

$$a = bq + r$$

$$0 \leq r \leq b$$

Algorithm :- An algorithm is a series of well defined steps which gives a procedure for solving a type of problem.

Lemma :- A lemma is a proven statement used for proving another statement.

Euclid's Division Algorithm: To obtain HCF of two positive integers, say c and d, with c > d, follow the steps below:

Step 1: Apply Euclid's Division lemma, to c and d. So, we find whole number q and r such that
$c = dq + r$, $0 \leq r \leq d$

Step 2: If r = 0, d is the HCF of c and d. If r ≠ 0, apply the division lemma to d and r

Step 3: Continue the process till r = 0. The divisor at this stage will be the required HCF.

Exercise : 1.1

1. Use Euclid's division algorithm to find the HCF of :

(i) 135 and 225

Sol:- Using Euclid's division algorithm

$d = 135$, $c = 225$; $c > d$

$c = dq + r$

$225 = 135 \times 1 + 90$

Now,

Again apply Euclid's division algorithm, ~~here~~

$135 = 90 \times 1 + 45$

Now again,

apply Euclid's division algorithm

$90 = 45 \times 2 + 0$

Hence,

HCF of 135 & 225 = 45

(ii) 196 and 38220

Sol:- Given, $c = 38220$, $d = 196$ and $c > d$

Using Euclid's division algorithm

$c = dq + r$

$38220 = 196 \times 195 + 0$

Hence, HCF of 196 and 38220 = 196

(iii) 867 and 255

Sol:- Given, $c = 867$, $d = 255$ and $c > d$

Using Euclid's division algorithm

$c = dq + r$

$867 = 255 \times 3 + 102$

Now,

Using Euclid's division algorithm

$255 = 102 \times 2 + 51$

Now again,

Using Euclid's division algorithm

$102 = 51 \times 2 + 0$

Hence,

HCF of 867 and 255 = 51

2. Show that any positive odd integer is of the form $6q+1$, or $6q+3$, or $6q+5$, where q is some integer.

Sol: We apply Euclid division algorithm

$b = 6$

Now

$a = 6q + r$, $r = 0, 1, 2, 3, 4, 5$

Then,

$r = 0$

$a = 6q + 0$

$a = 6q$ — Even

When $r = 1$

$a = 6q + 1$ —— odd

When $r = 2$

$a = 6q + 2$ —— even

$a = 2(3q + 1)$

When $r = 3$

$a = 6q + 3$

$a = 3(2q + 1)$ —— odd

When $r = 4$

$a = 6q + 4$

$a = 2(3q + 2)$ —— even

When $r = 5$

$a = 6q + 5$ —— odd

Hence,

We can say that $6q + 1$, $6q + 3$ and $6q + 5$ are odd integers.

3. An army contigent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Sol:: Given, total members in an army contigent = 616

members in an army band = 32

maximum number of columns = ?

Then,

Using Euclid's division algorithm.

$b = 32$; $a = 616$

HCF of (616, 32)

$a = bq + r$

$616 = 32 \times 19 + 8$

Again,

apply Euclid's division algorithm

$32 = 8 \times 4 + 0$

Hence,

Maximum number of columns = 8

4. Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Sol:- Given, $b = 3$

Then,

$r = 0, 1, 2$

Now,

Case I: Using $r = 0$, apply Euclid's division algorithm

$a = 3q + 0$

$a = 3q$

Squaring both sides

$(a)^2 = (3q)^2$

$a^2 = 9q^2$

$a^2 = 3(3q^2)$

$a^2 = 3m$ where $m = 3q^2$

Then,

Case II: Using $r = 1$, apply Euclid's division lemma

$a = 3q + 1$

$(a)^2 = (3q + 1)^2$

$a^2 = (3q)^2 + 2 \times 3q \times 1 + (1)^2$

$a^2 = 9q^2 + 6q + 1$

$a^2 = 3(3q^2 + 2q) + 1$

$a^2 = 3m + 1$, where $m = 3q^2 + 2q$

Case III : Using $r = 2$, we apply Euclid's division lemma

$a = 3q + 2$

$(a)^2 = (3q + 2)^2$

$a^2 = (3q)^2 + 2 \times 3q \times 2 + (2)^2$

$a^2 = 9q^2 + 12q + 4$

$a^2 = 3(3q^2 + 4q + 1) + 1$

$a^2 = 3m + 1$, where $m = 3q^2 + 4q + 1$

Hence,

Square of any positive integer is either in the form $3m$ or $3m + 1$ for some integer m.

Exercise - 1.2

1. Express each number as a product of its prime factors:

(i) 140

Sol: LCM of $140 = 2 \times 2 \times 5 \times 7$
$= 2^2 \times 5 \times 7$

(ii) 156

Sol: LCM of $156 = 2 \times 2 \times 3 \times 13$
$= 2^2 \times 3 \times 13$

(iii) 3825

Sol: LCM of $3825 = 3 \times 3 \times 5 \times 5 \times 17$
$= 3^2 \times 5^2 \times 17$

(iv) 5005

Sol: LCM of $5005 = 5 \times 7 \times 11 \times 13$

(v) 7429

Sol: LCM of $7429 = 17 \times 19 \times 23$

2. Find the LCM and HCF of the following pairs of integers and verify that LCM X HCF = product of the two numbers.

(i) 26 and 91

Sol. Given, I^{st} number = 26
II^{nd} number = 91

LCM = $2 \times 7 \times 13 = 182$
HCF = 13
Now,
Prove:

LCM × HCF = Product of two numbers
$2 \times 7 \times 13 \times 13 = 26 \times 91$
$182 \times 13 = 2366$
$2366 = 2366$
Proved

(ii) 510 and 92

Sol: Given, I^{st} number = 510
II^{nd} number = 92

LCM = $2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$
HCF = 2
Now,
Prove:

LCM × HCF = I^{st} number × II^{nd} number
$23460 \times 2 = 510 \times 92$
$46920 = 46920$
Proved

(iii) 336 and 54

Sol: Given, I^{st} number = 336
II^{nd} number = 54

Now,
LCM = $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 = 3024$
HCF = 6

Prove :

LCM × HCF = I^{st} number × II^{nd} number
$3024 \times 6 = 336 \times 54$
$18144 = 18144$
Proved

3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21

Sol:- The prime factorisation of 12, 15 and 21 ;

$12 = 2 \times 2 \times 3$
$15 = 3 \times 5$
$21 = 3 \times 7$

Then,
LCM = $2 \times 2 \times 3 \times 5 \times 7 = 420$
HCF = 3

(ii) 17, 23 and 29

Sol: The prime factorisation of 17, 23 and 29;

$17 = 1 \times 17$
$23 = 1 \times 23$
$29 = 1 \times 29$

Then,

LCM = $17 \times 23 \times 29 = 11339$

HCF = 1

(iii) 8, 9 and 25

Sol: The prime factorisation of 8, 9 and 25;

$8 = 2 \times 2 \times 2$

$9 = 3 \times 3$

$25 = 5 \times 5$

Then,

LCM = $2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 1800$

HCF = 1

4. Given that HCF (306, 657) = 9, find LCM (306, 657).

Sol: Given, I^{st} number = 306

II^{nd} number = 657

HCF = 9

LCM = ?

Then,

$\therefore$ LCM $\times$ HCF = I^{st} number $\times$ II^{nd} number

LCM $\times 9 = 306 \times 657$

LCM = $\dfrac{\overset{34}{\cancel{306}} \times 657}{\underset{1}{\cancel{9}}}$

LCM = 34×657

LCM = 22338

5. Check whether 6^n can end with the digit 0 for any natural number n.

Sol: To check, 6^n can end with 0 or not
Then,
For any number that ends with 0 must have a multiplied by 5

Prime factor of $6 = 2 \times 3$
So,
it cannot end with 0.

6 Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Sol: For $7 \times 11 \times 13 + 13$
$= 7 \times 11 \times 13 (1+1)$
$= 1001 \times 2 = 1 \times 1001 \times 2$
Here,
$7 \times 11 \times 13 + 13$ has more than 2 factors
Hence,
it is composite number
Now,
for, $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$
$= 7 \times 6 \times 4 \times 3 \times 2 \times 1 \times 5 + 5$
$= 7 \times 6 \times 4 \times 3 \times 2 \times 1 \times 5 (1+1)$
$= 7 \times 6 \times 4 \times 3 \times 2 \times 5 \times 2$
$= 7 \times 6 \times 4 \times 3 \times 20$
Here,
$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ has more than 2 factor
Hence,
it is composite number.

7 There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they

both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Sol:- Given, time taken by Sonia to complete one round = 18 min.
time taken by Ravi to complete one round = 12 min

Then,
time when they meet again at starting point;

LCM of 18 and 12 = $2 \times 2 \times 3 \times 3 = 36$ minutes.

Exercise : 1.3

1. Prove that $\sqrt{5}$ is irrational.

Sol:: Let $\sqrt{5}$ is a rational number
Then,

$$\sqrt{5} = \frac{a}{b} \quad , b \neq 0$$

So,

$$a = \sqrt{5}\, b$$

Squaring both sides, we get

$$(a)^2 = (\sqrt{5}\, b)^2$$

$$a^2 = 5b^2 \quad \text{--- (i)}$$

5 divides a^2
5 divides a

So,
we can write
$a = 5c$ (where c is any integer)

Putting value of a in eqn (i)

$$(5c)^2 = 5b^2$$

$$25c^2 = 5b^2$$

$$5c^2 = b^2$$

It means 5 divides b^2 and 5 divides b

So,
5 is a common factor of both a and b

Then,
Our assumption that $\sqrt{5}$ is rational is wrong.

Hence,
$\sqrt{5}$ is an irrational number.

2. Prove that $3 + 2\sqrt{5}$ is irrational.

Sol:- Let $3+2\sqrt{5}$ be rational number.

So,

$$3+2\sqrt{5} = \frac{a}{b} \quad , \text{where } b \neq 0$$

$$2\sqrt{5} = \frac{a}{b} - 3$$

$$2\sqrt{5} = \frac{a-3b}{b}$$

$$\sqrt{5} = \frac{a-3b}{2b}$$

Since,

$\frac{a-3b}{2b}$ is a rational number

Therefore,

$\sqrt{5}$ is also a rational number

Thus,

Our assumption is wrong

Hence,

$3+2\sqrt{5}$ is an irrational number.

3. Prove that following are irrationals.

(i) $\frac{1}{\sqrt{2}}$

Sol:- Let $\frac{1}{\sqrt{2}}$ be a rational number

So,

$$\frac{1}{\sqrt{2}} = \frac{a}{b} \quad , \text{where } b \neq 0$$

$b = \sqrt{2}a$

Squaring both sides, we get.

$(b)^2 = (\sqrt{2}a)^2$

$b^2 = 2a^2$ ——— (i)

So,

2 divides b^2

2 divides b

So,

we can write

$b = 2c$, where c is any integer

Putting value of b in eqⁿ (i)

$(2c)^2 = 2a^2$

$4c^2 = 2a^2$

$2c^2 = a^2$

It means 2 divides a^2 and also a

So,

2 is common factor of both a and b

So,

Our assumption that $\frac{1}{\sqrt{2}}$ is a rational number is wrong.

Hence,

$\frac{1}{\sqrt{2}}$ is an irrational number.

(ii) $7\sqrt{5}$

Sol:- Let $7\sqrt{5}$ be a rational number

So,

$7\sqrt{5} = \frac{a}{b}$, where $b \neq 0$

$\sqrt{5} = \frac{a}{7b}$

Since, $\frac{a}{7b}$ is a rational number

Therefore,

$\sqrt{5}$ is also a rational number

Thus,

Our assumption is wrong.

Hence,

$7\sqrt{5}$ is an irrational number.

(iii) $6+\sqrt{2}$

Sol: Let $6+\sqrt{2}$ is a rational number

So,

$$6+\sqrt{2} = \frac{a}{b}, \text{ where } b \neq 0$$

$$\sqrt{2} = \frac{a}{b} - 6$$

$$\sqrt{2} = \frac{a-6b}{b}$$

Since,

$\frac{a-6b}{b}$ is a rational number

Therefore,

$\sqrt{2}$ is also a rational number

Thus,

Our assumption is wrong.

Hence,

$6+\sqrt{2}$ is an irrational number.

Exercise: 1.4

1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) $\frac{13}{3125} \Rightarrow \frac{13}{5^5}$

$\because$ denominator has 5^m expansion
$\therefore$ Rational number is terminating decimal expansion

(ii) $\frac{17}{8} \Rightarrow \frac{17}{2^3}$

$\because$ denominator has 2^n expansion
$\therefore$ Given Rational no. is terminating.

(iii) $\frac{64}{455} \Rightarrow \frac{64}{5 \times 7 \times 13}$

$\because$ Denominator has $5 \times 7 \times 13$ expansion
$\therefore$ Given Rational no. is non-terminating repeating

(iv) $\frac{15}{1600} \Rightarrow \frac{15}{2^6 \times 5^2}$

$\because$ Denominator has $2^n 5^m$ expansion.
$\therefore$ Given Rational no. is terminating

(v) $\frac{29}{343} \Rightarrow \frac{29}{7^3}$

∵ Denominator has 7^3 expansion
∴ Given Rational number is non terminating repeating

(vi) $\frac{23}{2^3 5^2}$

Sol: ∵ ~~Given~~ Denominator has $2^n 5^m$ expansion
∴ Given Rational no. is terminating

(vii) $\frac{129}{2^2 5^7 7^5}$

Sol: ∵ Denominator does not has $2^n 5^m$ expansion
∴ Given Rational no. is non terminating repeating

(viii) $\frac{6}{15} \Rightarrow \frac{\cancel{6}^2}{\cancel{3} \times 5} = \frac{2}{5}$

∵ Denominator ~~does not~~ has $2^n 5^m$ expansion
∴ Given Rational no. is ~~non~~ terminating ~~repeating~~

(ix) $\frac{35}{50} \Rightarrow \frac{35}{2 \times 5^2}$

∵ Denominator has $2^n 5^m$ expansion
∴ Given rational no. is terminating

(x) $\frac{77}{210} \Rightarrow \frac{77}{2 \times 3 \times 5 \times 7}$

∵ Denominator does not has $2^n 5^m$ expansion
∴ Given rational no. is non terminating repeating

2. Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Sol: The decimal expansions of terminating decimal expansion in Question 1 are :-

(i) $\frac{13}{3125} \Rightarrow 0.00416$

(ii) $\frac{17}{8} \Rightarrow \frac{17 \times 5^3}{2^3 \times 5^3} = \frac{2125}{(2\times5)^3} = \frac{2125}{10^3} = \frac{2125}{1000} = 2.125$

(iv) $\frac{15}{1600} \Rightarrow \frac{15}{2^6 \times 5^2} = \frac{15 \times 5^4}{2^6 \times 5^2 \times 5^4} = \frac{9375}{2^6 \times 5^{2+4}} = \frac{9375}{2^6 \times 5^6}$

$= \frac{9375}{(2\times5)^6} = \frac{9375}{10^6} = 0.009375$

(vi) $\frac{23}{2^3 5^2} \Rightarrow \frac{23 \times 5}{2^3 \times 5^2 \times 5} = \frac{115}{2^3 \times 5^3} = \frac{115}{(2\times5)^3} = \frac{115}{10^3} = 0.115$

(viii) $\frac{6}{15} \Rightarrow \frac{\cancel{6}\,2}{\cancel{15}\,5} = \frac{2\times2}{5\times2} = \frac{4}{10} = 0.4$

(ix) $\frac{35}{50} \Rightarrow \frac{35\times2}{2\times5^2\times2} = \frac{70}{2^2\times5^2} = \frac{70}{(2\times5)^2} = \frac{70}{10^2} = 0.7$

3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form $\frac{p}{q}$, what does you say about prime factor of q?

(i) 43.123456789

Sol: $\because$ Given no. is terminating
$\therefore$ It is a rational number

Sol: $\dfrac{43123456789}{1000000000} = \dfrac{43123456789}{2^{10} \times 5}$

Hence,
$\frac{q}{r}$ has prime factor 2 and 5 only

(ii) 0.120120012000120000...

Sol: $\because$ Given no. is non-terminating non-repeating
$\therefore$ It is irrational number.

(iii) $43.\overline{123456789}$

Sol: $\because$ Given no. is non-terminating repeating
$\therefore$ It is rational number.
Now,
$x = 43.123456789123456789 - - -$ —— (I)
Then,
Multiply by 1000000000 in eqn (I)

$1000000000\,x = 43123456789.123456789 - - -$ —— (II)
Now,
Subtract eqn (I) from (II)

$1000000000\,x = 43123456789.123456789 - - -$
$x = 43.123456789 - - -$

$999999999x = 43123456746.000000000$

$$x = \frac{4312345674 6}{99999999}$$

Hence,

q has prime factor other than 2 and 5

Summary

★ Euclid's division lemma: Given positive integers a and b, there exists whole numbers q and r satisfying

$a = bq + r$, $0 \le r \le b$

The Fundamental Theorem of Arithmetic:

★ Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factor occur.

★ If p is a prime number and divides a^2, then p divides a, where a is a positive integer.

★ Let x be a decimal number whose decimal expansion terminates. Then we can express x in the form $\frac{p}{q}$, where p and q are co-prime, and the prime factorisation of q is of the form $2^m 5^n$, where m & n are non-negative integers