Chapter 2

Polynomials

Polynomial:- A combination of constant and variable is known as polynomial.

Types of Polynomials:-

1. **According to degree**:-

(i) **Linear Polynomial**:- A polynomial of degree 1 is called a linear polynomial.

(ii) **Quadratic Polynomial**:- A polynomial of degree 2 is called a Quadratic Polynomial.

(iii) **Cubic Polynomial**:- A polynomial of degree 3 is called a cubic polynomial.

(iv) **Biquadratic Polynomial**:- A polynomial of degree 4 is called a Biquadratic polynomial.

2. **According to term**:-

(i) **Monomial**:- A polynomial of one term is called monomial.

(ii) **Binomial**:- A polynomial of two term is called binomial

(iii) **Trinomial**:- A polynomial of three term is called trinomial

★ ~~(iv)~~ Polynomial :- A polynomial of more than three terms is called a polynomial

Formulae :-

(i) $\alpha + \beta = \frac{-b}{a}$

(ii) $\alpha\beta = \frac{c}{a}$

(iii) $\alpha + \beta + \gamma = \frac{-b}{a}$

(iv) $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

(v) $\alpha\beta\gamma = \frac{-d}{a}$

Division Algorithms for Polynomials :-

If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that

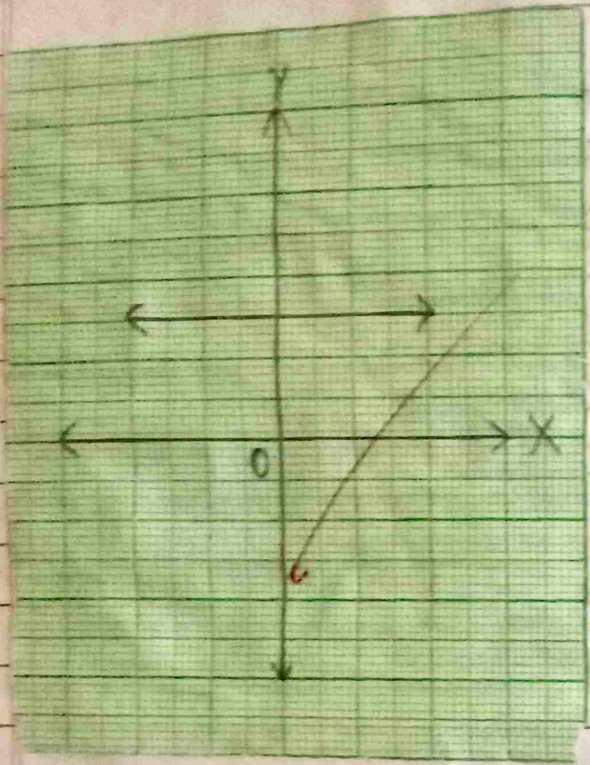
$$p(x) = g(x) \times q(x) + r(x)$$

where $r(x) = 0$ or degree of $r(x)$ < degree of $g(x)$

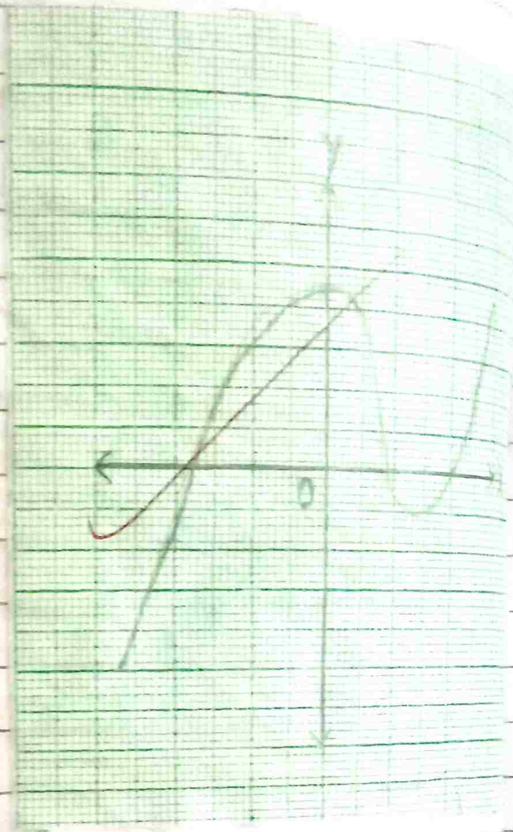
Exercise 2.1

1. The graphs of $y = p(x)$ are given in Fig 2.10 below, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.

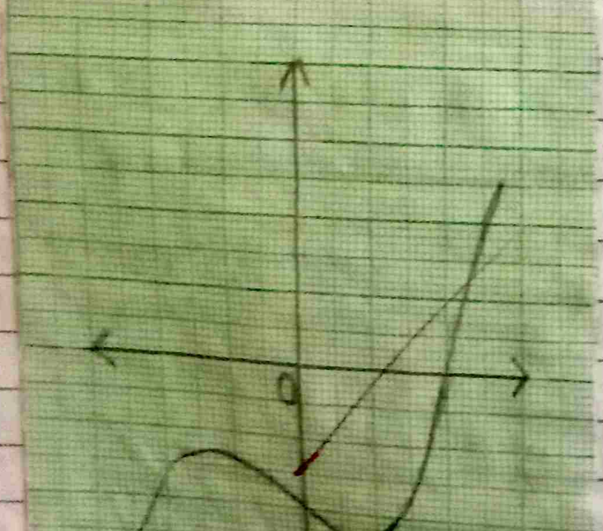
(i)



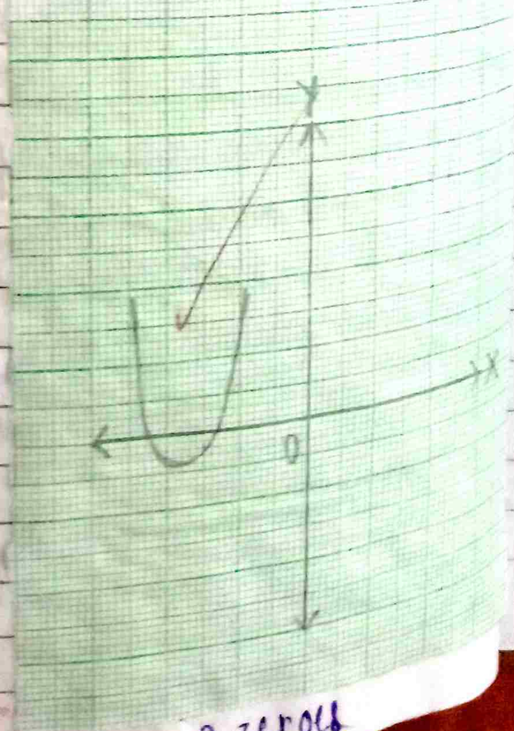
No zeroes



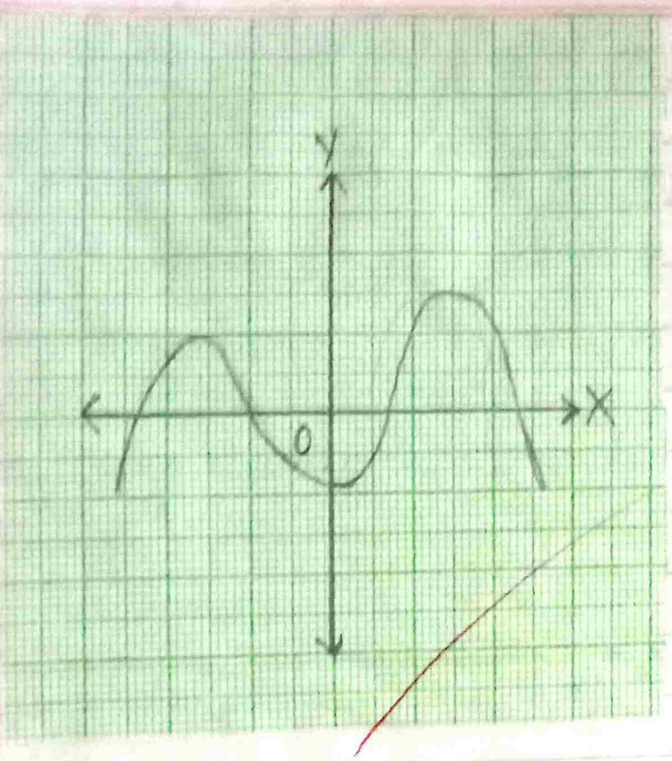
3 zeroes



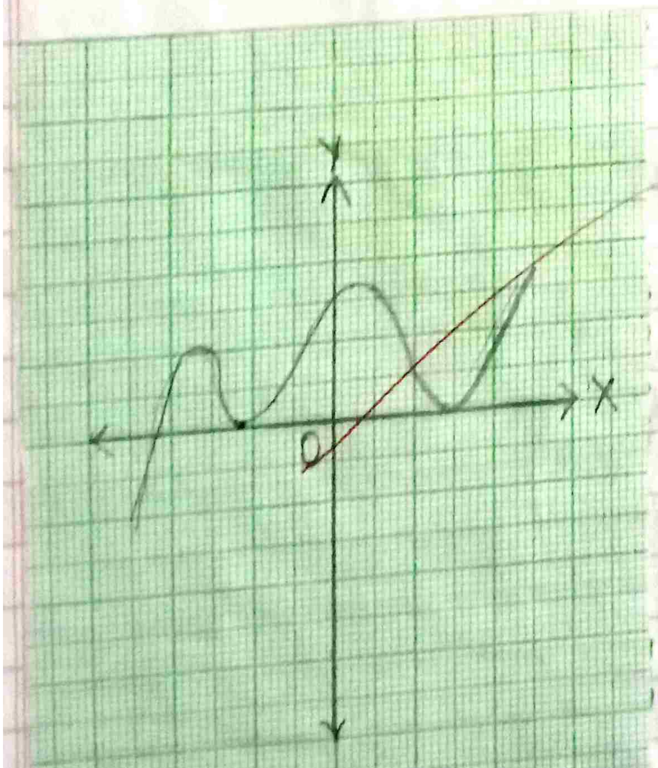
1 zero



2 zeroes



4 zeroes



3 zeroes

Exercise 2.2

1 Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$

Sol: $x^2 + 2x - 4x - 8$

$x(x+2) - 4(x+2)$

$(x+2)(x-4)$

Zeroes $\Rightarrow$ (i) $x + 2 = 0$

$x = -2$

(ii) $x - 4 = 0$

$x = 4$

Now,

To verify relationship between zeroes and coefficients:

(i) Sum of zeroes $= -2 + 4 = 2$

Sum of coefficients $(\alpha + \beta) = \frac{-b}{a} = \frac{-(-2)}{1} = 2$

(ii) Product of zeroes $= -2 \times 4 = -8$

Product of coefficients $(\alpha\beta) = \frac{c}{a} = \frac{-8}{1} = -8$

(ii) $4s^2 - 4s + 1$

Sol: $4s^2 - 2s - 2s + 1$

$2s(2s-1) - 1(2s-1)$
$(2s-1)(2s-1)$

Zeroes :- (i) $2s - 1 = 0$
$2s = 1$
$s = \frac{1}{2}$

(ii) $2s - 1 = 0$
$2s = 1$
$s = \frac{1}{2}$

Now,
To verify relationship between zeroes and coefficient

(i) Sum of zeroes $= \frac{1}{2} + \frac{1}{2} = \frac{1+1}{2} = \frac{\cancel{2}^1}{\cancel{2}_1} = 1$

Sum of coefficients $(\alpha + \beta) = \frac{-b}{a} = \frac{-(-4)}{4} = \frac{\cancel{4}^1}{\cancel{4}_1} = 1$

(ii) Product of zeroes $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Product of coefficients $(\alpha\beta) = \frac{c}{a} = \frac{1}{4}$

(iii) $6x^2 - 3 - 7x$

Sol:- $6x^2 - 7x - 3$
$6x^2 + 2x - 9x - 3$
$2x(3x+1) - 3(3x+1)$
$(3x+1)(2x-3)$

Zeroes ⇒ (i) $3x+1 = 0$

$3x = -1$

$x = \frac{-1}{3}$

(ii) $2x-3 = 0$

$2x = 3$

$x = \frac{3}{2}$

Now,

To verify relationship between zeroes and coefficients

(i) ~~Zeroes ⇒ (i)~~ Sum of zeroes $= \frac{-1}{3} + \frac{3}{2} = \frac{-2+9}{6} = \frac{7}{6}$

Sum of coefficients $(\alpha + \beta) = \frac{-b}{a} = \frac{-(-7)}{6} = \frac{7}{6}$

(ii) Product of zeroes $= \frac{-1}{3} \times \frac{3}{2} = \frac{-\cancel{3}^{1}}{\cancel{6}_{2}} = \frac{-1}{2}$

Product of coefficients $(\alpha\beta) = \frac{c}{a} = \frac{-\cancel{3}^{1}}{\cancel{6}_{2}} = \frac{-1}{2}$

(iv) $3x^2 - x - 4$

Sol: $3x^2 - x - 4$

$3x^2 + 3x - 4x - 4$

$3x(x+1) - 4(x+1)$

$(x+1)(3x-4)$

Zeroes ⇒ (i) $x+1 = 0$

$x = -1$

(ii) $3x - 4 = 0$

$3x = 4$

$x = \frac{4}{3}$

Now,

To verify relationship between zeroes and coefficients

(i) Sum of zeroes $= -1 + \frac{4}{3} = \frac{-3+4}{3} = \frac{1}{3}$

Sum of coefficients $(\alpha + \beta) = \frac{-b}{a} = \frac{-(-1)}{3} = \frac{1}{3}$

(ii) Product of zeroes $= -1 \times \frac{4}{3} = \frac{-4}{3}$

Product of coefficients $(\alpha\beta) = \frac{c}{a} = \frac{-4}{3}$

(v) $t^2 - 15$

Sol:- zeroes of $t^2 - 15$ are :

(i) $t^2 - 15 = 0$

$t^2 = 15$

$t = \sqrt{15}$

Now,

To verify relationship between zeroes and coefficients

(i) Sum of zeroes $= -\sqrt{15} + \sqrt{15} = 0$

Sum of coefficients $(\alpha+\beta) = \frac{-b}{a} = \frac{0}{1} = 0$

(ii) Product of coefficients $(\alpha\beta) = \frac{c}{a} = \frac{-15}{1} = -15$

Product of zeroes $= -\sqrt{15} \times \sqrt{15} = -15$

(vi) $4u^2 + 8u$

Sol: Zeroes of $4u^2+8u$ are:

(i) $4u^2 + 8u = 0$
$4u(u+2) = 0$
$u+2 = 0$
$u = -2$

Now,
To verify relationship between zeroes and coefficients

(i) Sum of zeroes $= -2+0 = -2$

Sum of coefficients $(\alpha+\beta) = \frac{-b}{a} = \frac{-8}{4} = -2$

(ii) ~~So~~ Product of zeroes $= -2 \times 0 = 0$

Product of coefficients $(\alpha\beta) = \frac{c}{a} = \frac{0}{4} = 0$

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4}$, -1

Sol:- Let zeroes of polynomial be 'a and β'
Then,

$a + \beta = \frac{1}{4}$; $a\beta = -1$

Now,

polynomial is :

$p(x) = x^2 - (a+\beta)x + a\beta$

$= x^2 - \left(\frac{1}{4}\right)x + (-1)$

$= x^2 - \frac{x}{4} - 1$

Multiply by 4 in p(x)

$p(x) = 4x^2 - \frac{x}{\cancel{4}} \times \cancel{4} - 1 \times 4$

$p(x) = 4x^2 - x - 4$

(ii) $\sqrt{2}$, $\frac{1}{3}$

Sol:- Let zeroes of polynomial be 'a and β'
Now,

$a + \beta = \sqrt{2}$; $a\beta = \frac{1}{3}$

Then,

$p(x) = x^2 - (a+\beta)x + a\beta$

"It's fine to celebrate success but it is more important to heed the lessons of failure." —Bill Gates

$$p(x) = x^2 - \sqrt{2}\,x + \frac{1}{3}$$

Multiply by 3 in p(x)

$$p(x) = 3x^2 - 3\sqrt{2}\,x + \frac{1}{\cancel{3}} \times \cancel{3}$$

$$p(x) = 3x^2 - 3\sqrt{2}\,x + 1$$

(iii) $0, \sqrt{5}$

Sol:- Let zeroes of polynomial be 'α & β'

Now,

$$\alpha + \beta = 0 \quad , \quad \alpha\beta = \sqrt{5}$$

Then,

$$p(x) = x^2 + (\alpha+\beta)x + \alpha\beta$$

$$p(x) = x^2 + 0x + \sqrt{5}$$

$$p(x) = x^2 + \sqrt{5}$$

(iv) $1, 1$

Sol:- Let zeroes of polynomial be 'α & β'

Now,

$$\alpha + \beta = 1 \quad ; \quad \alpha\beta = 1$$

Then,

$$p(x) = x^2 - (\alpha+\beta)x + \alpha\beta$$

$$= x^2 - (1)x + 1$$

$$= x^2 - x + 1$$

(v) $\frac{-1}{4}, \frac{1}{4}$

Sol:- Let zeroes of polynomial be 'α & β'

Now,

$\alpha + \beta = -\frac{1}{4}$; $\alpha\beta = \frac{1}{4}$

Then,

$p(x) = x^2 - (\alpha+\beta)x + \alpha\beta$

$= x^2 - \left(\frac{-1}{4}\right)x + \frac{1}{4}$

$= x^2 + \frac{1}{4}x + \frac{1}{4}$

Multiply by 4 in $p(x)$

$p(x) = 4x^2 + \frac{1}{\cancel{4}}x \times \cancel{4} + \frac{1}{\cancel{4}} \times \cancel{4}$

$p(x) = 4x^2 + x + 1$

(vi) 4, 1

Sol:- Let zeroes of polynomial be 'α & β'

Now,

$\alpha + \beta = 4$; $\alpha\beta = 1$

Then,

$p(x) = x^2 - (\alpha+\beta)x + \alpha\beta$

$p(x) = x^2 - 4x + 1$

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Exercise = 2.3

1. Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x)$ $x^2 - 2$

Sol:- $x^2 - 2 \overline{) x^3 - 3x^2 + 5x - 3} (x - 3$

$$\begin{array}{l} x^3 \qquad\qquad - 2x \\ - \qquad\qquad\quad + \\ \hline -3x^2 + 7x - 3 \\ -3x^2 \qquad\quad + 6 \\ + \qquad\qquad\quad - \\ \hline 7x - 9 \end{array}$$

Hence,

Quotient = $x - 3$

Remainder = $7x - 9$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

Sol:- $x^2 - x + 1 \overline{) x^4 - 3x^2 + 4x + 5} (x^2 + x - 3$

$$\begin{array}{l} x^4 - x^3 + x^2 \\ - \quad + \quad - \\ \hline x^3 - 4x^2 + 4x \\ x^3 - x^2 + x \\ - \quad + \quad - \\ \hline -3x^2 + 3x + 5 \\ -3x^2 + 3x - 3 \\ + \quad - \quad + \\ \hline 8 \end{array}$$

Hence,

Quotient = $x^2 + x - 3$

Remainder = 8

(iii) $p(x) = x^4 - 5x + 6$, $g(x) = 2 - x^2$

Sol:-
$$
\begin{array}{r|l}
-x^2+2 & x^4 - 5x + 6 \quad (-x^2 - 2 \\
& x^4 - 2x^2 \\
& - \quad + \\
\hline
& 2x^2 - 5x + 6 \\
& 2x^2 \quad - 4 \\
& - \quad + \\
\hline
& -5x + 10
\end{array}
$$

Hence,

Quotient : $-x^2 - 2$

Remainder : $-5x + 10$

2. Check whether the first polynomial is a factor of second polynomial by dividing second polynomial by the first polynomial:

(i) $t^2 - 3$, $2t^4 + 3t^3 - 2t^2 - 9t - 12$

Sol:-
$$
\begin{array}{r|l}
t^2-3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \quad (2t^2 + 3t + 4 \\
& 2t^4 \quad - 6t^2 \\
& - \quad + \\
\hline
& 3t^3 + 4t^2 - 9t \\
& 3t^3 \quad - 9t \\
& - \quad + \\
\hline
& 4t^2 \quad - 12 \\
& 4t^2 \quad - 12 \\
& - \quad + \\
\hline
& 0 \\
\hline
\end{array}
$$

$\because$ Remainder = 0

$\therefore$ $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) x^2+3x+1 , $3x^4+5x^3-7x^2+2x+2$

Sol:- $x^2+3x+1 \,\big)\, 3x^4+5x^3-7x^2+2x+2 \,\big(\, 3x^2-4x+2$

$$
\begin{array}{r}
3x^4+9x^3+3x^2 \\
- \quad - \quad - \\
\hline
-4x^3-10x^2+2x \\
-4x^3-12x^2-4x \\
+ \quad + \quad + \\
\hline
2x^2+6x+2 \\
2x^2+6x+2 \\
- \quad - \quad - \\
\hline
0 \\
\hline
\end{array}
$$

$\because$ Remainder $= 0$

$\therefore$ x^2+3x+1 is a factor of $3x^4+5x^3-7x^2+2x+2$

(iii) x^3-3x+1, $x^5-4x^3+x^2+3x+1$

Sol:- $x^3-3x+1 \,\big)\, x^5-4x^3+x^2+3x+1 \,\big(\, x^2 \cancel{+} -1$

$$
\begin{array}{r}
x^5-3x^3+x^2 \\
- \quad + \quad - \\
\hline
-x^3 \quad +3x+1 \\
-x^3 \quad +3x-1 \\
+ \quad - \quad + \\
\hline
2
\end{array}
$$

$\because$ Remainder $\neq 0$

$\therefore$ x^3-3x+1 is a factor of $x^5-4x^3+x^2+3x+1$

3. Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Sol: Given, $p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$

Zeroes = $\sqrt{\frac{5}{3}}$, $-\sqrt{\frac{5}{3}}$

So,

$\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = x^2 - \frac{5}{3}$ is a factor of $p(x)$

Other zeroes = ?

Then,

$$
\begin{array}{r|l}
x^2 - \frac{5}{3} \;\big)\; 3x^4 + 6x^3 - 2x^2 - 10x - 5 & 3x^2 + 6x + 3 \\
3x^4 \qquad\quad - 5x^2 \qquad\qquad & \\
- \qquad\qquad + \qquad\qquad & \\
\hline
6x^3 + 3x^2 - 10x \quad & \\
6x^3 \qquad\quad - 10x \quad & \\
- \qquad\qquad + \qquad & \\
\hline
3x^2 - 5 & \\
3x^2 - 5 & \\
- \quad + & \\
\hline
0 &
\end{array}
$$

So,

$3x^4 + 6x^3 - 2x^2 - 10x - 5 = \left(x^2 - \frac{5}{3}\right)\left(3x^2 + 6x + 3\right)$

By splitting $3x^2 + 6x + 3$, we get

$3x^2 + 6x + 3$

$3x^2 + 3x + 3x + 3$

$3x(x+1) + 3(x+1)$

$(3x+3)(x+1)$

$3(x+1)(x+1)$

So, Other zeroes = $-1, -1$

Hence,

Zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5 = \sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1, -1$

4. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

Sol:- Given, $p(x) = x^3 - 3x^2 + x + 2$

$q(x) = x - 2$

$r(x) = -2x + 4$

$g(x) = ?$

As we know that,

$$p(x) = g(x) \times q(x) + r(x)$$

$$x^3 - 3x^2 + x + 2 = g(x) \times (x-2) + (-2x+4)$$

$$x^3 - 3x^2 + x + 2 = g(x) \times (x-2) - 2x + 4$$

$$x^3 - 3x^2 + x + 2 + 2x - 4 = g(x) \times (x-2)$$

$$x^3 - 3x^2 + 3x - 2 = g(x) \times (x-2)$$

$$\frac{x^3 - 3x^2 + 3x - 2}{x - 2} = g(x)$$

Then,

$$g(x) = x^2 - x + 1$$

5. Give examples of polynomials $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) $\deg p(x) = \deg q(x)$

Ans: $p(x) = 2x^2 - 2x + 14$, $g(x) = 2$, $q(x) = x^2 - x + 7$, $r(x) = 0$

(ii) $\deg q(x) = \deg r(x)$

Ans: $p(x) = x^3 + x^2 + x + 1$, $g(x) = x^2 - 1$, $q(x) = x + 1$, $r(x) = 2x + 2$

(iii) ~~deg(r)x~~ $\deg r(x) = 0$

Ans: $p(x) = x^3 + 2x^2 - x + 2$, $g(x) = x^2 - 1$, $q(x) = x + 2$, $r(x) = 4$

Summary

1. Polynomials of degree 1, 2 and 3 are called linear, quadratic and cubic polynomials respectively.

A quadratic polynomial in x with real coefficients is of the form $ax^2 + bx + c$, where a, b and c are real numbers with $a \neq 0$

A quadratic polynomial can have at most 2 zeroes and a cubic polynomial can have at most 3 zeroes.

If α and β are the zeroes of the quadratic polynomials $ax^2 + bx + c$, then

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

If α, β and γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$, then

$$\alpha + \beta + \gamma = \frac{-b}{a}, \quad \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} \text{ and}$$

$$\alpha\beta\gamma = \frac{-d}{a}$$